Sinyaller ve Sistemler

“Temel Sinyaller ve Fonksiyonlar”

Dr. Cahit Karakuş, 2020

Temel Sinyaller

Elementary signals

- Exponential signals
- Sinusoidal signals
- Hiperbolik Sinyaller
- Logaritmik fonkisyonlar
- Çko değişkenli fonksiyonlar
- Step function
- Rectangular pulse
- Impulse function
- Ramp function
- Polinomlar

Sinyal Analizi (2 – Boyutlu)

- Sinyalin kendisi
- Türevi
- İntegrali
- Limit
- Fourier Analizi
- Laplace ve Z dönüşümleri
- Yorumlanması

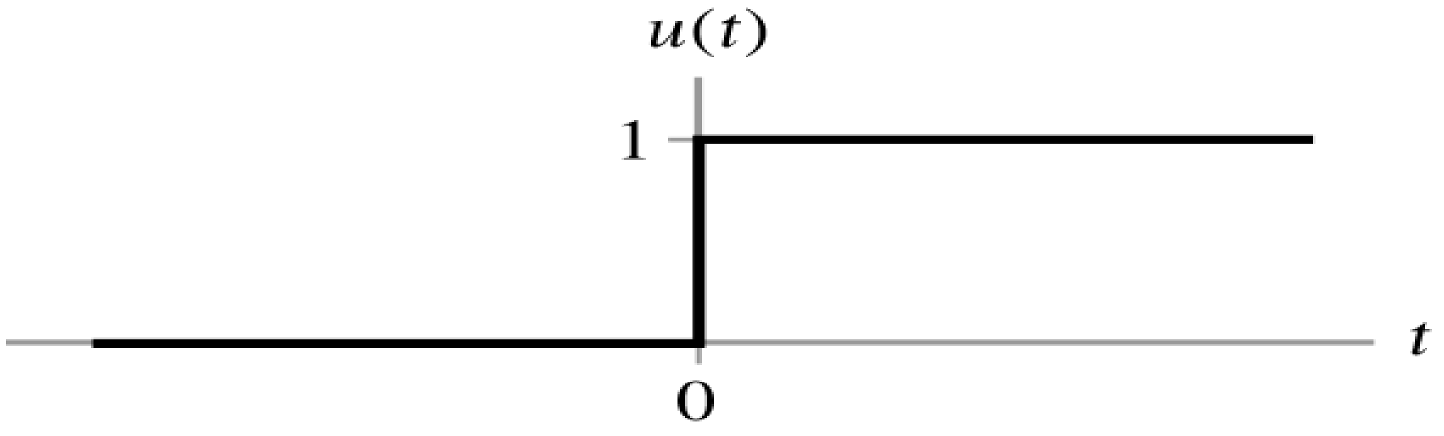
The elementary functions include

Constant functions: 2, e, π
Powers of x, x^2, x^3
Roots of x, $\sqrt{x}$
Exponential functions: e^x

Logarithms: log(x)
Trigonometric functions: sin(x), cos(x), tan(x)
Inverse trigonometric functions: arcsin(x), arccos(x), arctan(x)
Hyperbolic functions: sinh(x), cosh(x), tanh(x)
Inverse hyperbolic functions: arcsinh(x), arccosh(x),arc tanh(x)

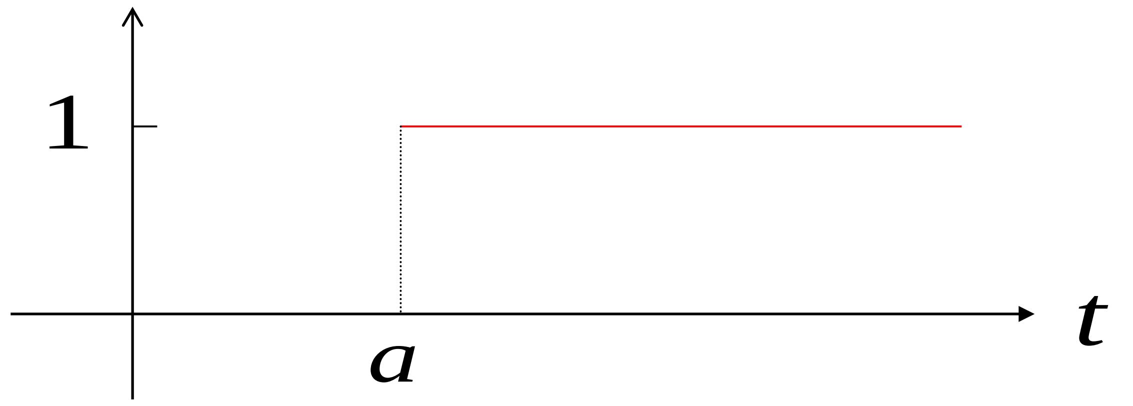
Step function

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

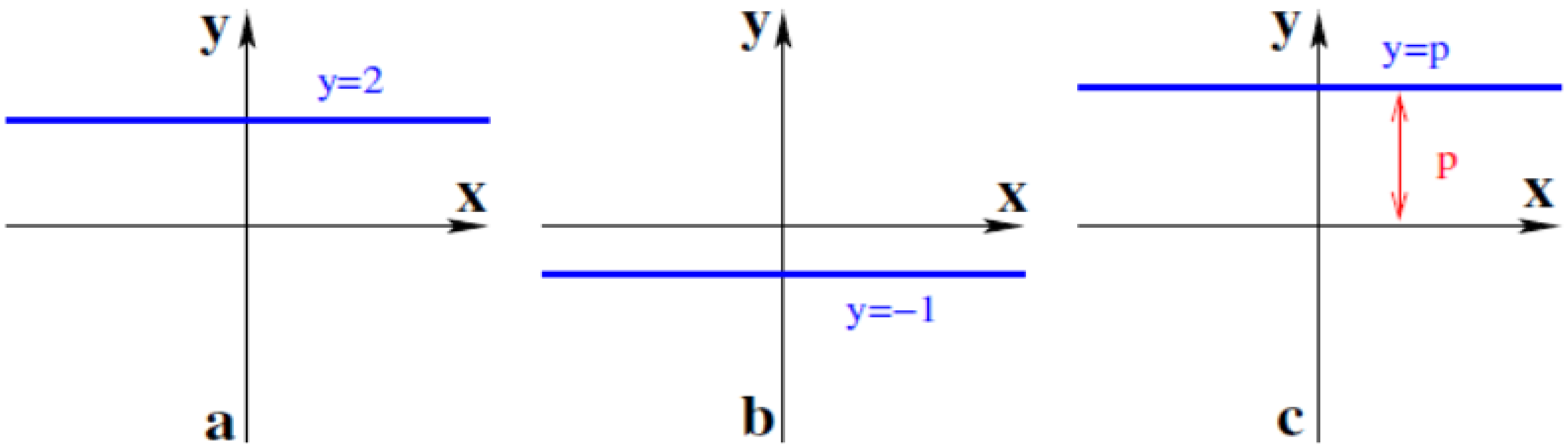


Shift a

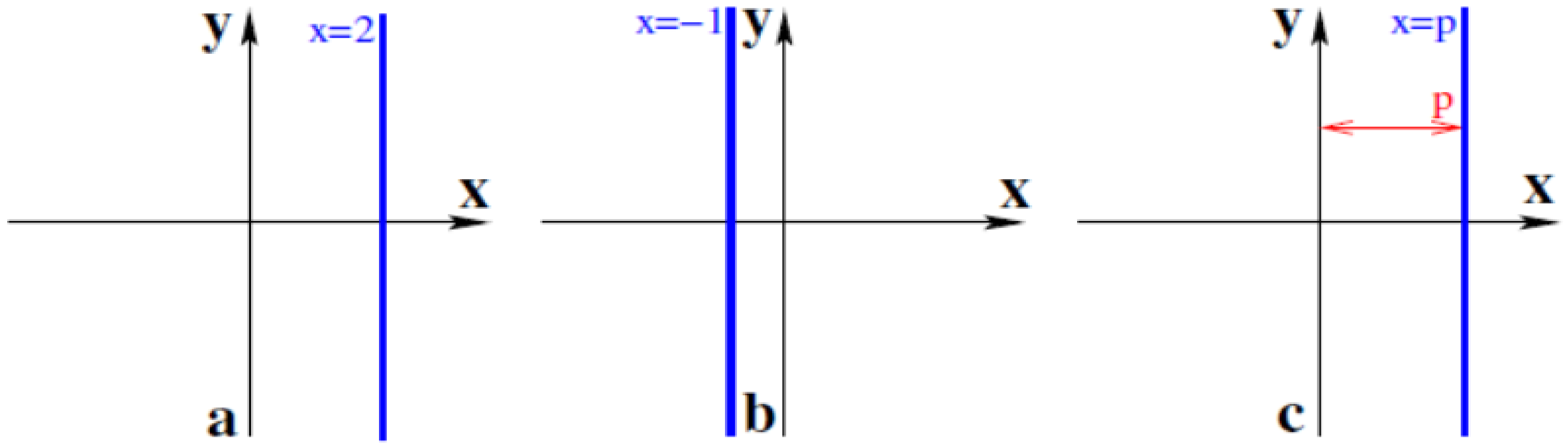
$$u(t-a) = \begin{cases} 1, & t > a \\ 0, & t < a \end{cases}$$



Doğru akım ya da gerilim sinyalleri

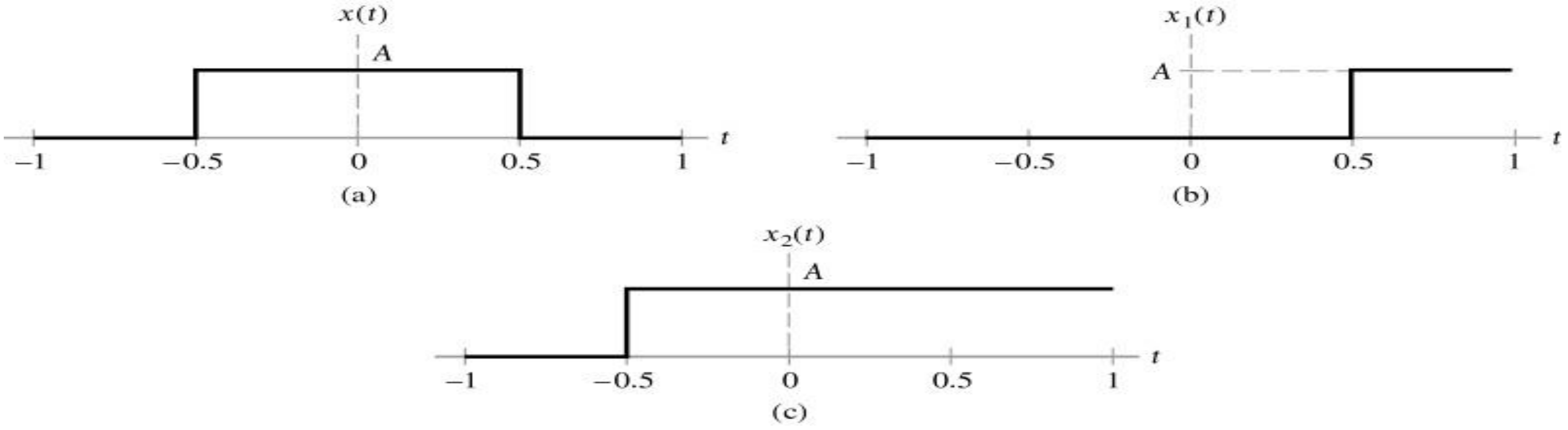


Equation y = p produces a horizontal line at the level p



Equation x = p produces a vertical line shifted by p from the y-axis

Rectangular pulse



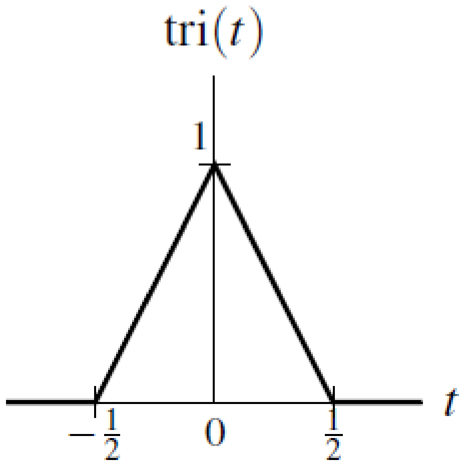
$$x(t) = \begin{cases} A, & -0.5 \leq |t| \leq 0.5 \\ 0, & otherwise \end{cases}$$

Triangular Function

- The **triangular function** (also called the unit-triangular pulse function), denoted tri, is defined as

$$\mathrm{tri}(t) = \begin{cases} 1 - 2|t| & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

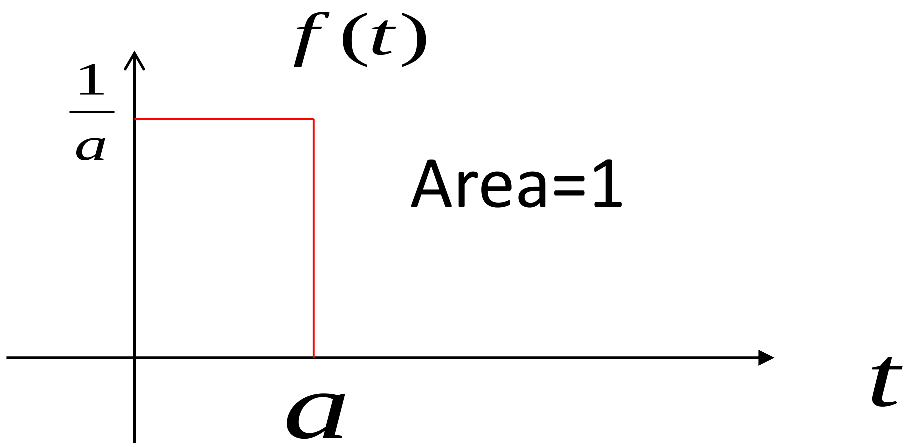
- A plot of this function is shown below.



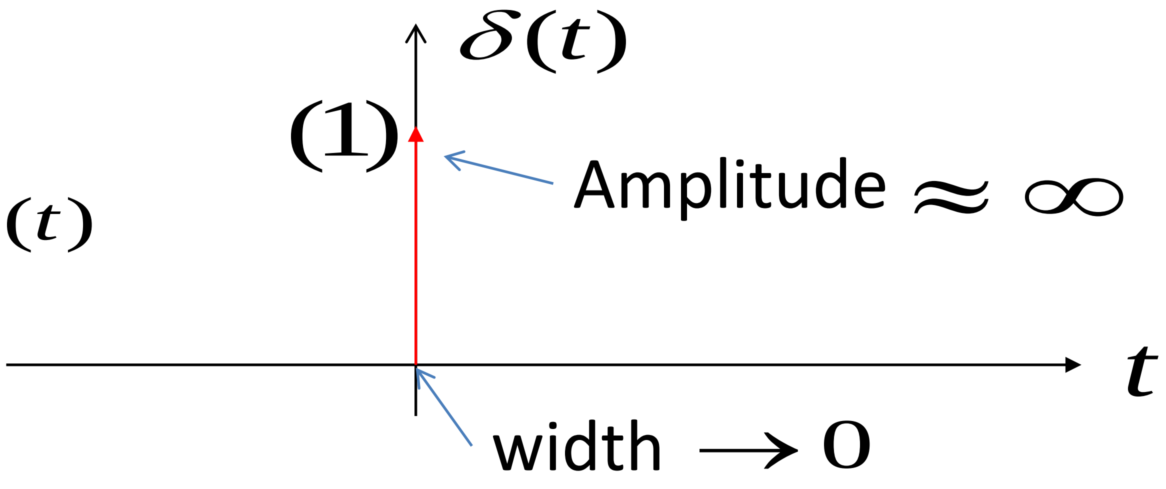
- Üçgen fonksiyon: Artan ve azalan rampa fonksiyonlarının bütünleşmesinden oluşur.
- Rampa fonkisyonu, y(t)=at+b, a ve b değerleri sabit değişkenledir.

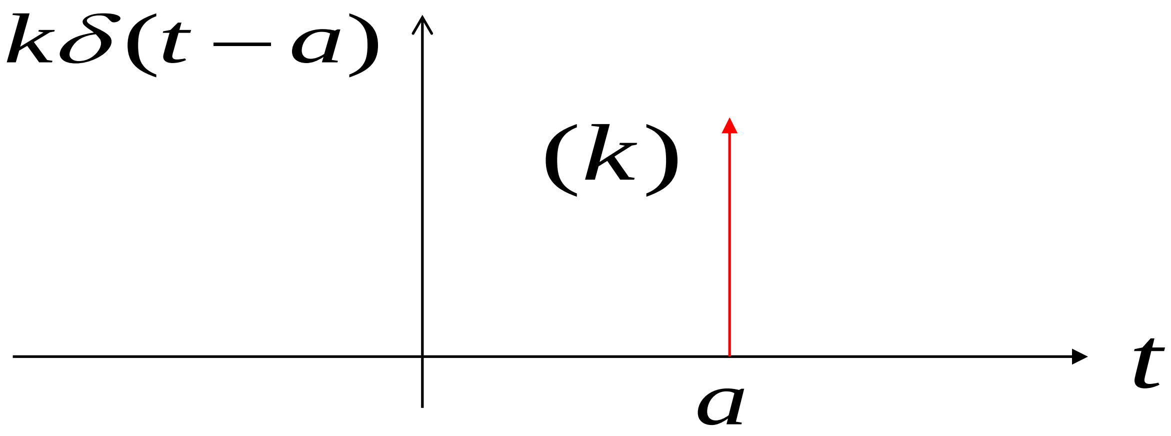
Unit impulse function

$$\delta(t) = \lim_{a \to 0} \frac{1}{a}[u(t) - u(t-a)]$$



$$\delta(t) = \frac{d}{dt} f(t)$$



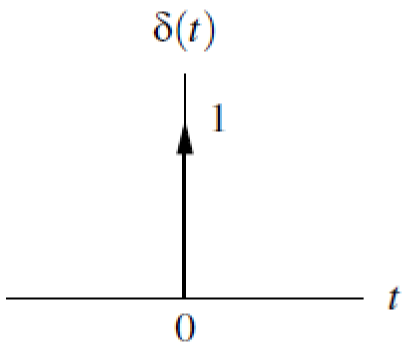


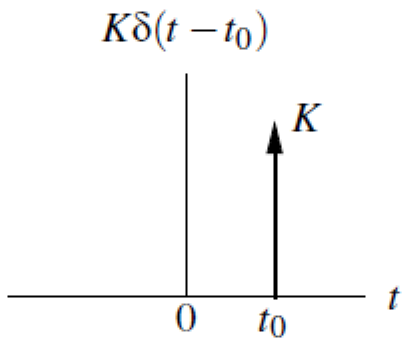
Unit-Impulse Function

- The **unit-impulse function** (also known as the **Dirac delta function** or **delta function**), denoted δ, is defined by the following two properties:

$$\delta(t) = 0 \quad \text{for } t \neq 0 \quad \text{and}$$
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1.$$

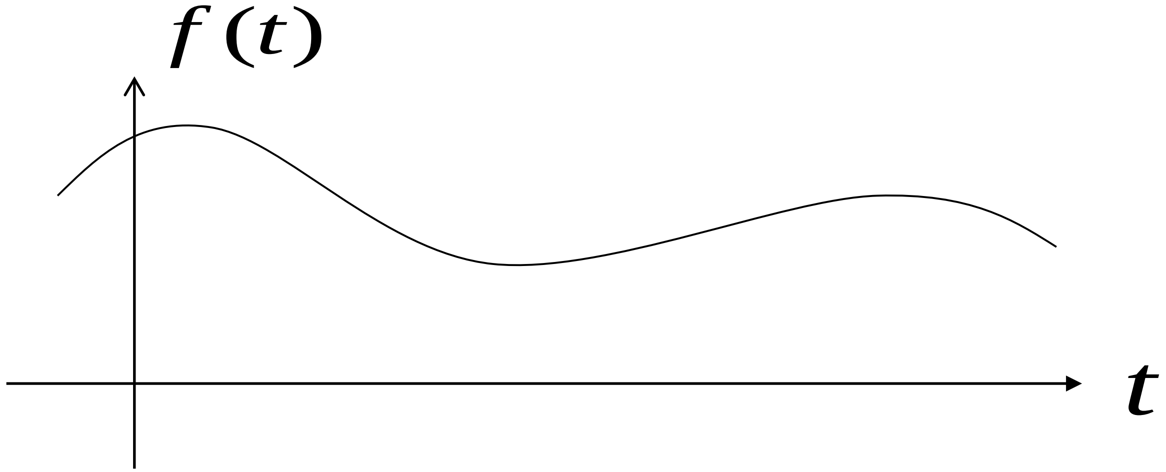
- Technically, δ is not a function in the ordinary sense. Rather, it is what is known as a *generalized function*. Consequently, the δ function sometimes behaves in unusual ways.
- Graphically, the delta function is represented as shown below.

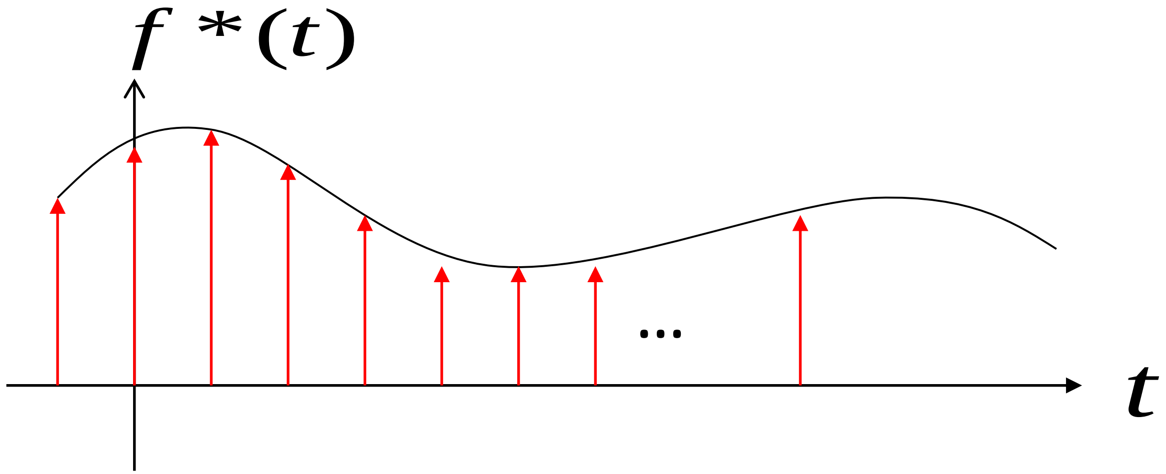




Örnek: t=15 saniyede f(t)=20birim. Geriye kalan tüm t saniyelerde f(t)=0birim ise bu fonksiyon ne olarak adlandırılır?

Sampling





$$f^*(t) = \sum_{n \rightarrow -\infty}^{\infty} f(t)\delta(t - nT)$$

Properties of the Unit-Impulse Function

- **Equivalence property.** For any continuous function x and any real constant t_0,

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0).$$

- **Sifting property.** For any continuous function x and any real constant t_0,

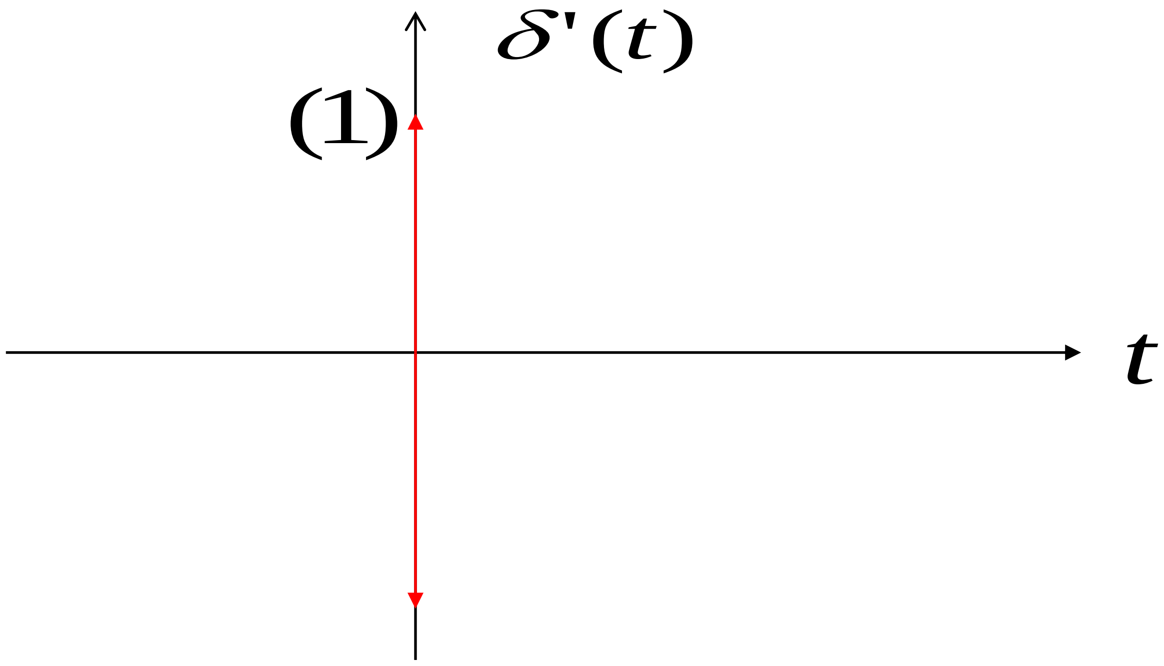
$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0).$$

- The δ function also has the following properties:

$$\delta(t) = \delta(-t) \quad \text{and}$$
$$\delta(at) = \frac{1}{|a|}\delta(t),$$

where a is a nonzero real constant.

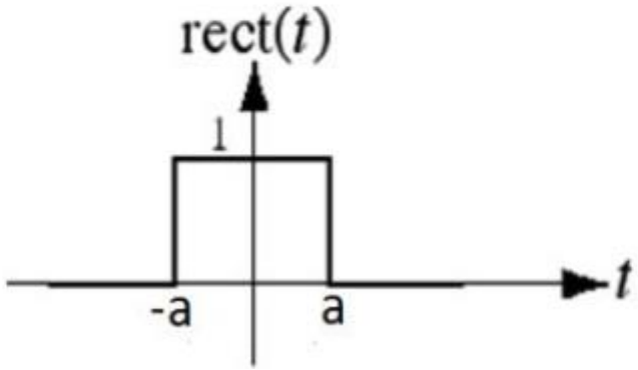
Unit doublet function $\delta'(t)$



Birim Diktörgen Adım Fonksiyonu

The unit rectangle or gate signal can be represented as combination of two shifted unit step signals as shown

$$rect(t) = u(t+a)-u(t-a)$$

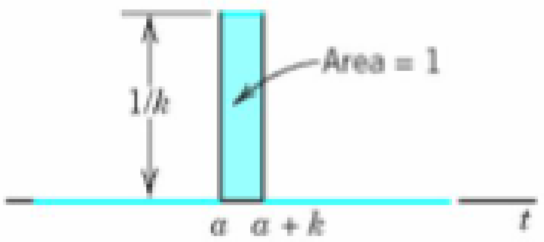


Darbe Fonksiyonu

Impulse Function

Define the function $f_k(t\text{-}a)$ as

$$f_k(t-a) = \begin{cases} 1/k & \text{if } a \leqq t \leqq a+k \\ 0 & \text{otherwise} \end{cases}$$



The function $f_k(t-a)$

In terms of unit step functions

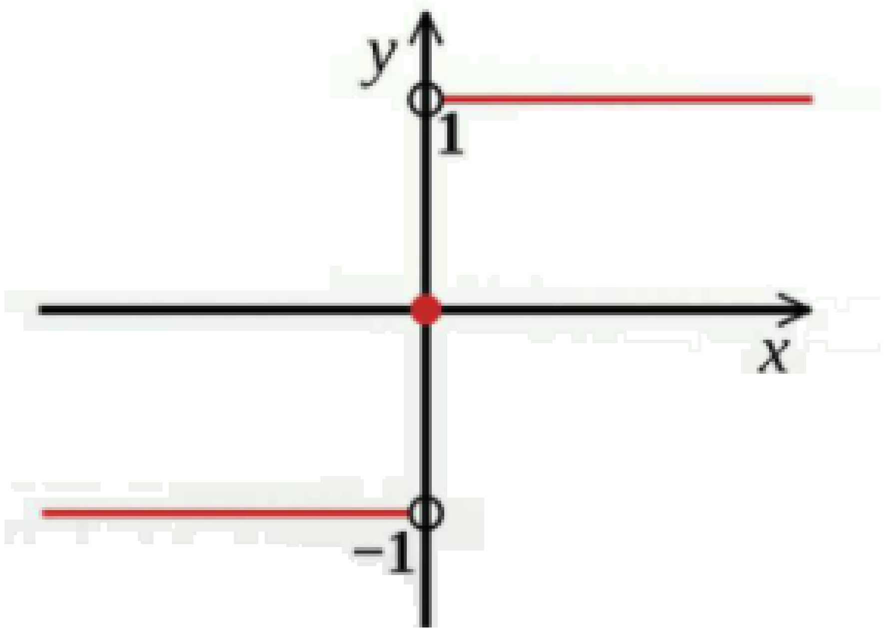
$$f_k(t-a) = \frac{1}{k}\left[u(t-a) - u(t-(a+k))\right]$$

Dirac delta function or unit impulse function

$$\delta(t-a) = \lim_{k \to 0} f_k(t-a).$$

sign function

$$\operatorname{sgn}(t) = \begin{cases} 1, & t \succ 0 \\ 0, & t = 0 \\ -1, & t \prec 0 \end{cases}$$

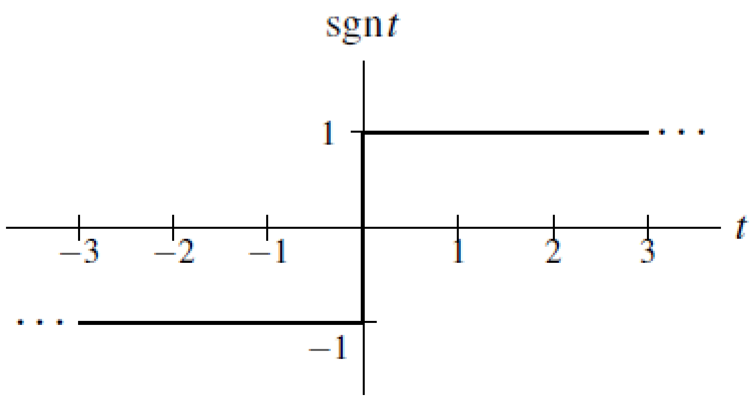


Signum Function

- The **signum function**, denoted sgn, is defined as

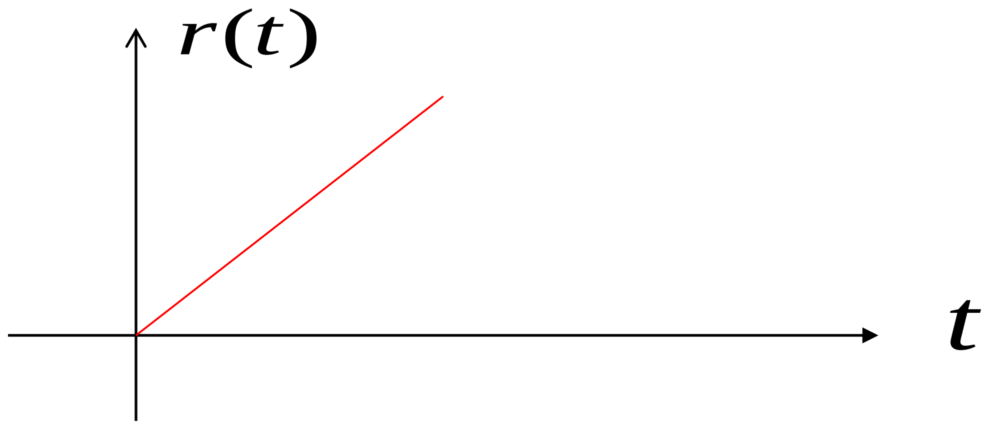
$$\operatorname{sgn} t = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t = 0 \\ -1 & \text{if } t < 0. \end{cases}$$

- From its definition, one can see that the signum function simply computes the ***sign*** of a number.
- A plot of this function is shown below.



Unit ramp signal

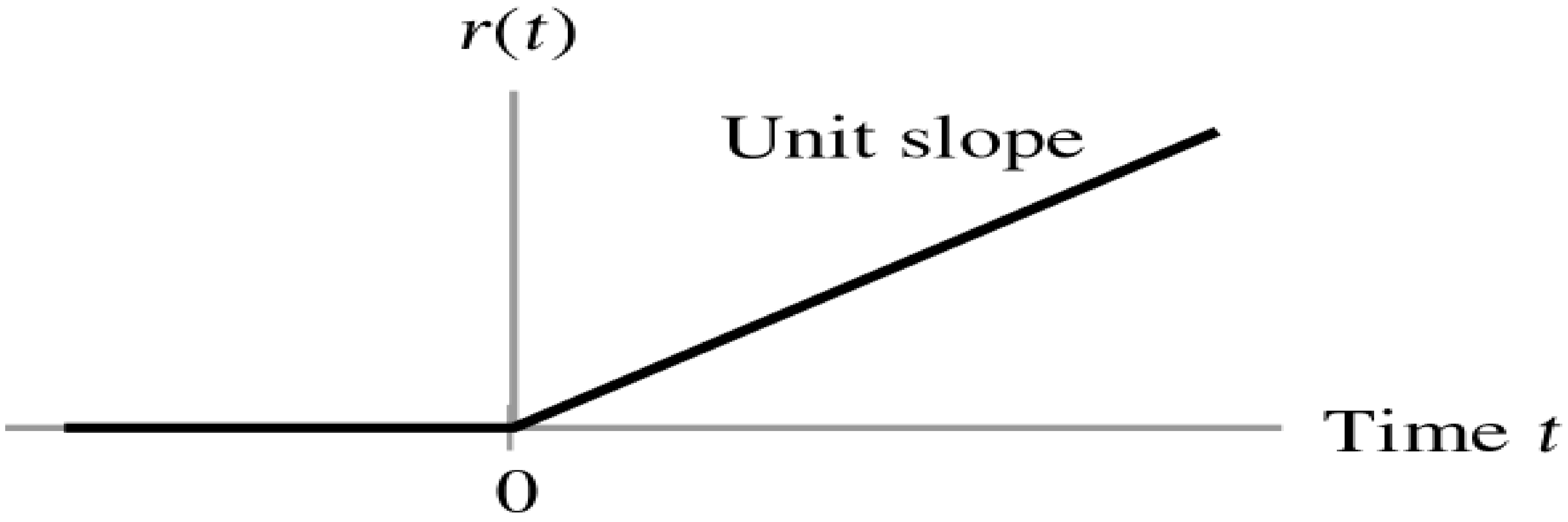
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t \prec 0 \end{cases}$$



$$u(t) = \frac{dr(t)}{dt} \qquad or \qquad r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

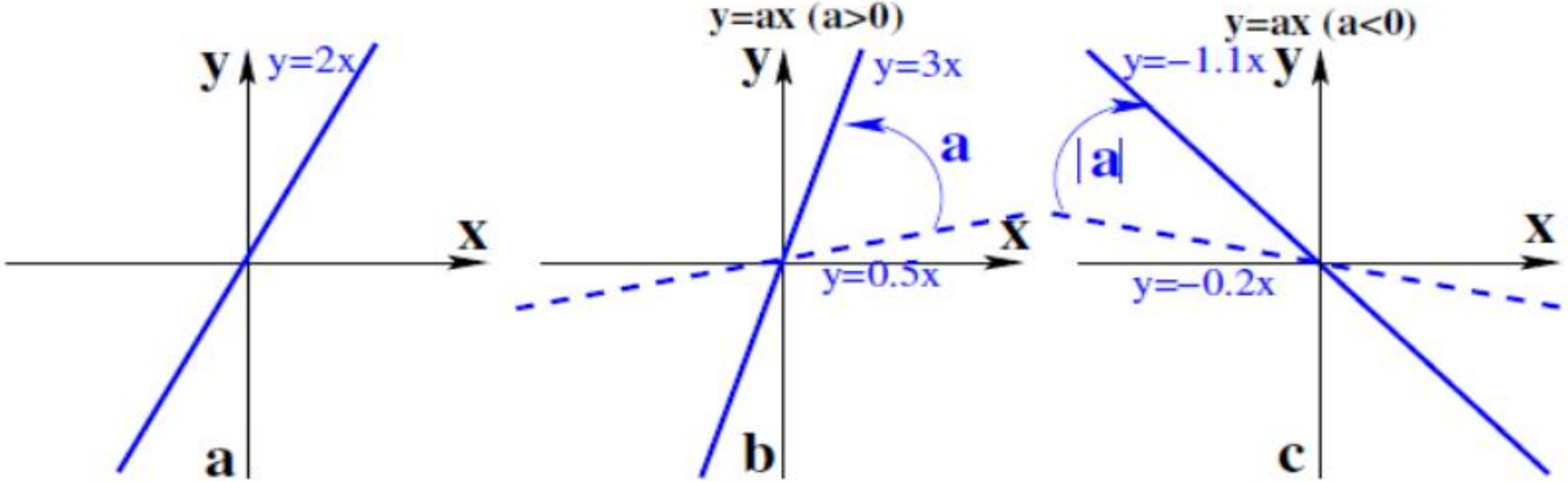
Ramp function

Genel anlamda rampa fonksiyonu, r(t)=at+b biçiminde yazılır. Burada, a ve b sabit değişkenlerdir.



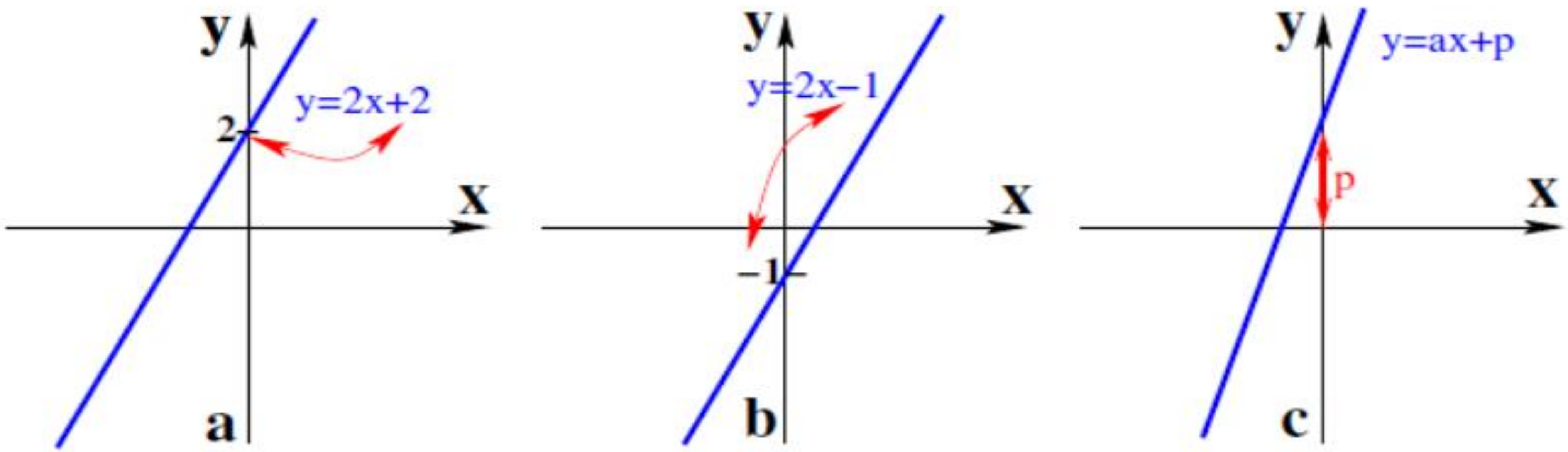
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Ramp function



Equation y = ax+ p (linear function)

Ramp function



The parameter p in y = ax+ p

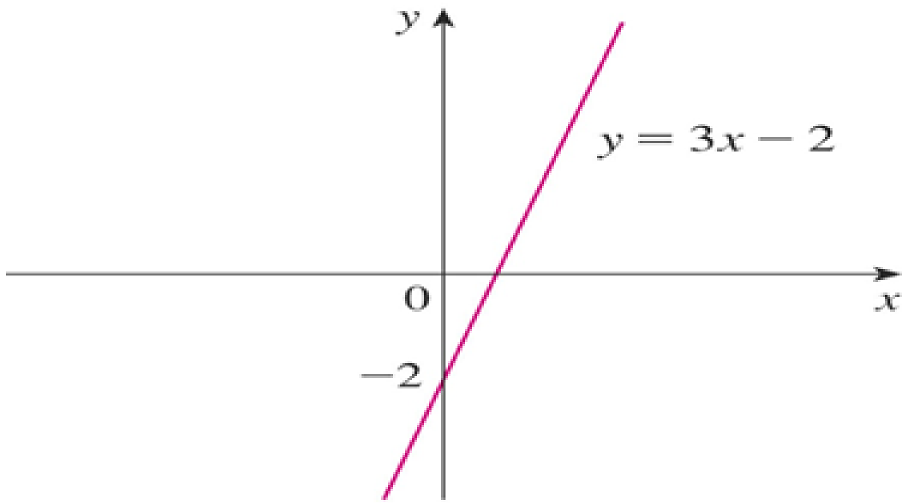
Doğrusal Modeller

- Y, x'in doğrusal bir fonksiyonu ise, fonksiyonun grafiğinin bir doğru olduğunu kastediyoruz.
- Böylece, bir doğrusal denklemin eğim-kesme noktalarında fonksiyon olarak yazabiliriz.

$$y = f(x) = mx + b$$

burada m, doğrunun eğimi ve b, y kesme noktasıdır.

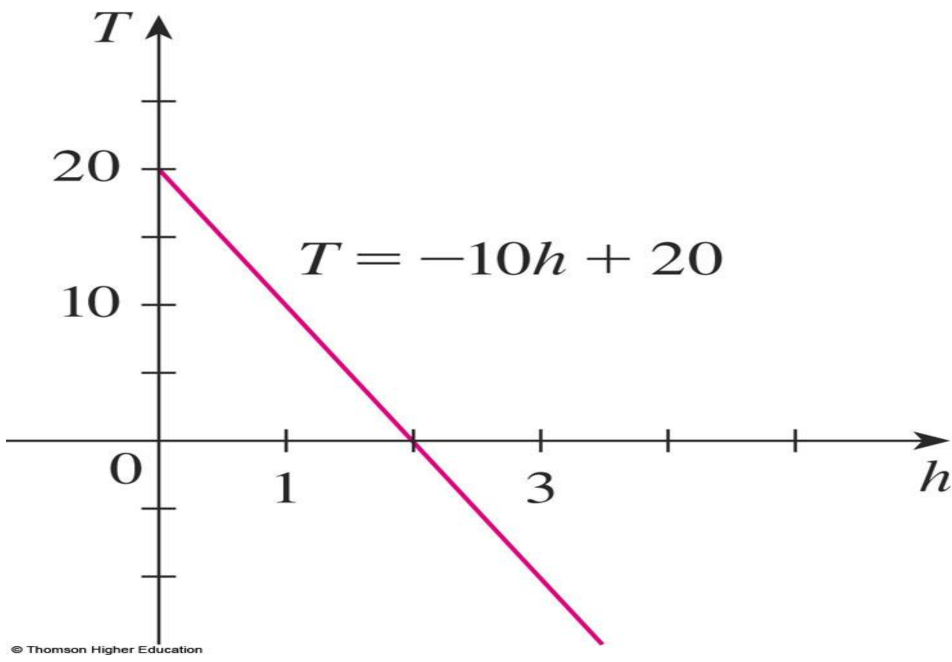
- Doğrusal fonksiyonların karakteristik bir özelliği, sabit bir oranda büyümeleridir.
- Örneğin, şekilde, f (x) = 3x - 2 doğrusal fonksiyonunun bir grafiği ve örnek değerler tablosu verilmiştir.
- 3 değeri grafiğinin eğimi, y'nin x'e göre değişim oranı olarak yorumlanabilir.
- X değeri 0.1 arttığında, f (x) değerinin 0.3 arttığına dikkat edin.
- Yani, f (x), x'in üç katı hızlı artar.



x	$f(x) = 3x - 2$
1.0	1.0
1.1	1.3
1.2	1.6
1.3	1.9
1.4	2.2
1.5	2.5

Doğrusal Modeller

Kuru hava yukarı doğru hareket ettikçe genişler ve soğur. Zemin sıcaklığı 20 ° C ve 1 km yükseklikteki sıcaklık 10 ° C ise, doğrusal bir modelin uygun olduğunu varsayarak sıcaklığı T (° C cinsinden) yüksekliğin (kilometre cinsinden) bir fonksiyonu olarak ifade edin. Fonksiyonun grafiğini çizin. Eğim neyi temsil ediyor? 2.5 km yükseklikte sıcaklık nedir?



T, h'nin doğrusal bir fonksiyonu olduğunu varsaydığımız için, T = mh + b yazabiliriz.

h = 0, yani 20 = m*0+b olduğunda, y kesme noktası b = 20'dir. Ayrıca, h = 1 olduğunda T = 10, m=-10 olur.

Gerekli doğrusal fonksiyon T = -10h + 20'dir.

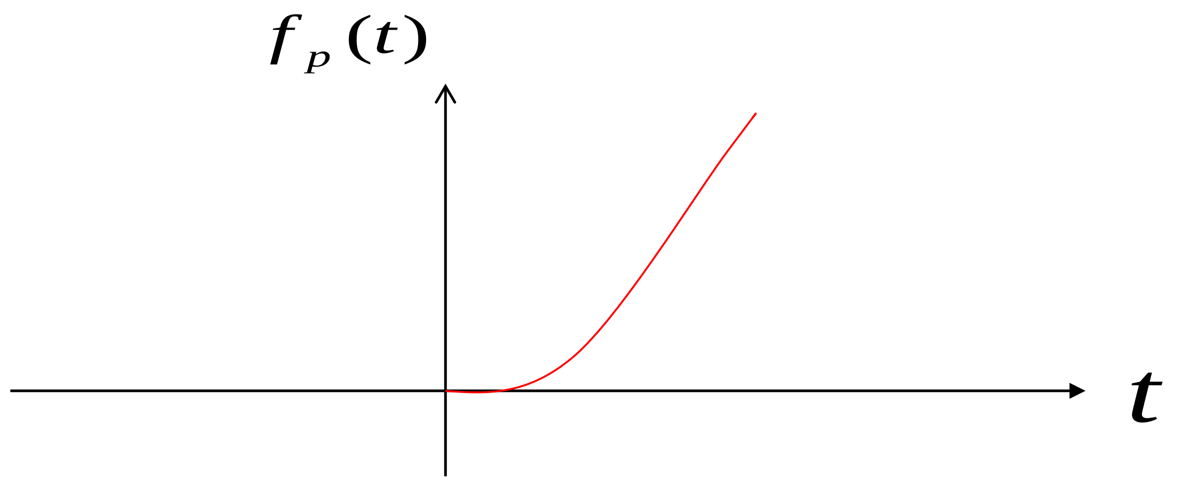
Eğim m = -10 ° C / km'dir.

Bu, yüksekliğe göre sıcaklık değişim oranını temsil eder.

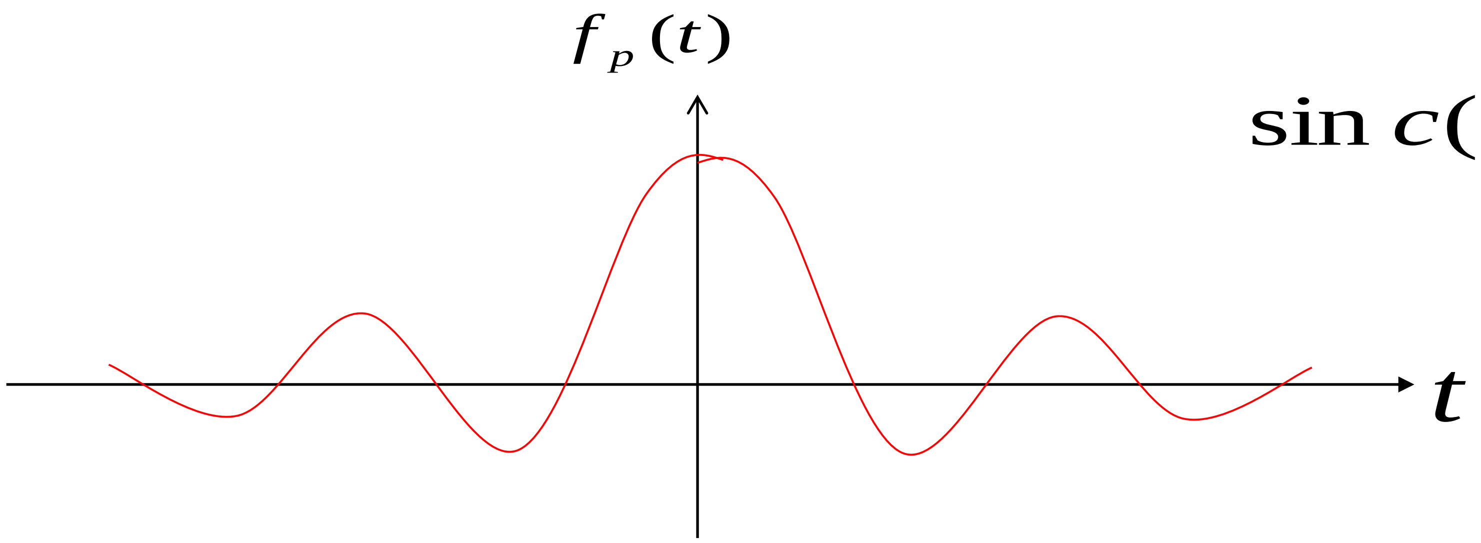
h = 2,5 km yükseklikte sıcaklık: T = -10 (2,5) + 20 = -5 ° C'dir.

Parabolic signal

$$f_p(t) = \begin{cases} t^2, & t \geq 0 \\ 0, & t \prec 0 \end{cases}$$



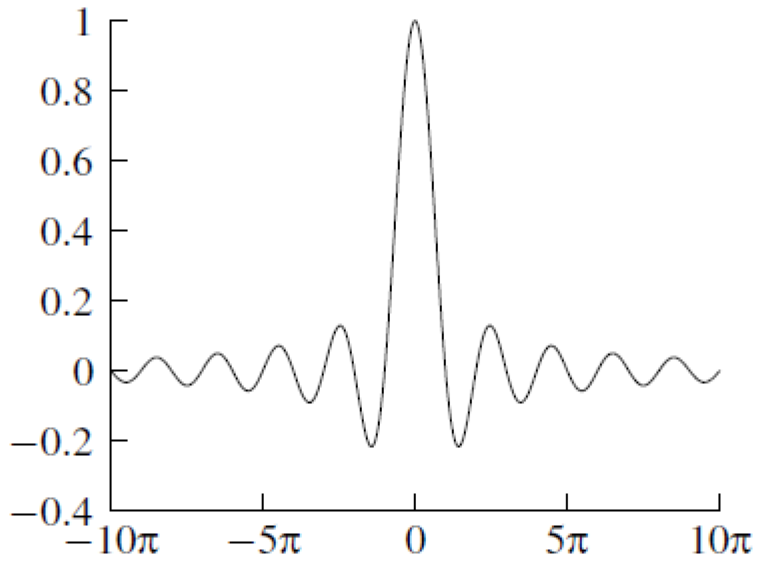
Sinc signal



$$\sin c(t) = \frac{\sin \pi t}{\pi t}$$

Cardinal Sine Function

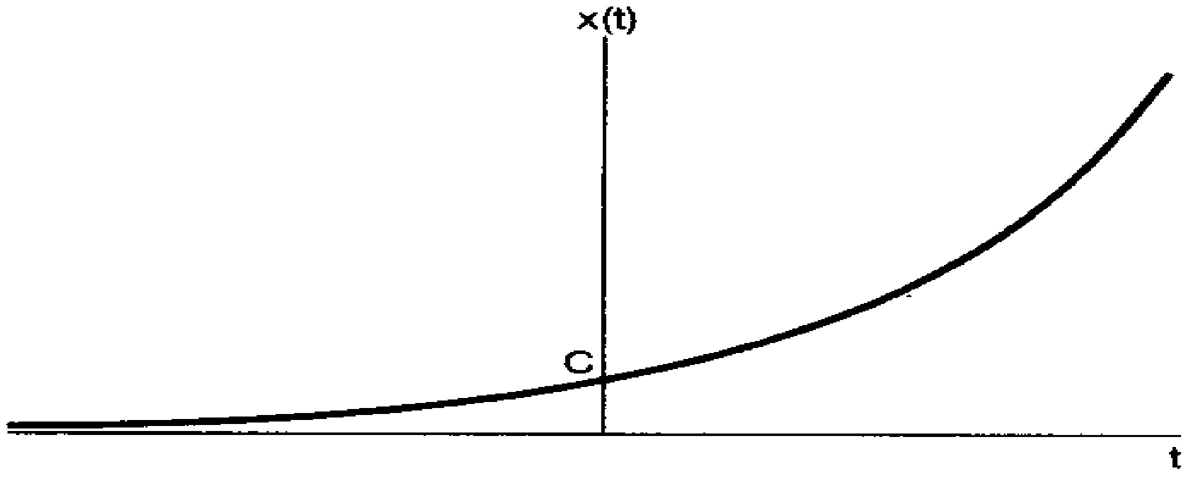
- The **cardinal sine** function, denoted sinc, is given by
$$\mathrm{sinc}(t) = \frac{\sin t}{t}.$$
- By l'Hopital's rule, $\mathrm{sinc}\,0 = 1$.
- A plot of this function for part of the real line is shown below. [Note that the oscillations in $\mathrm{sinc}(t)$ do not die out for finite t.]

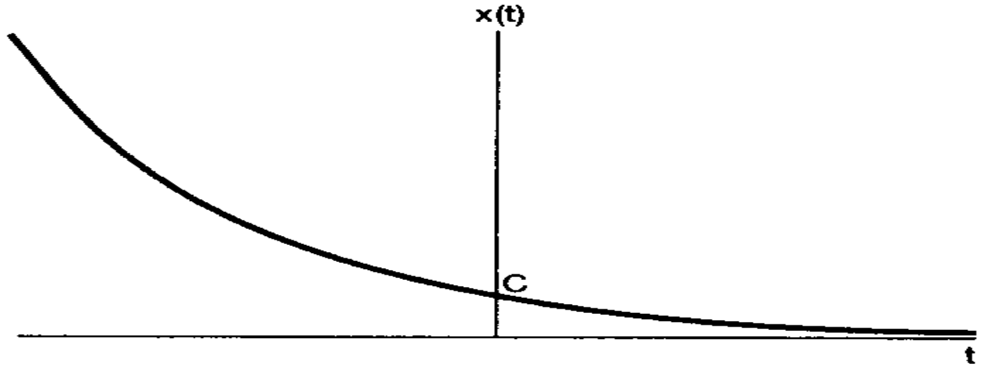


Exponential Signals

Continuous-Time Exponential Signals

- Real Exponential Signals (C and a are real)





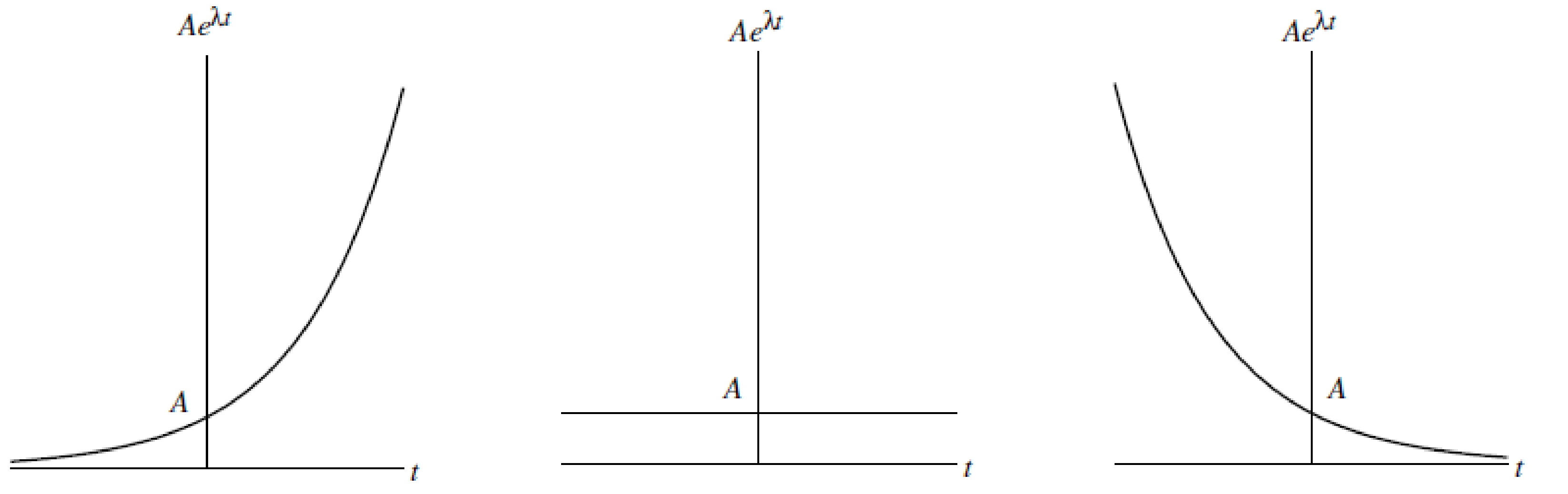
If $a>0$, $x(t)$ is a growing exponential

If $a<0$, $x(t)$ is a decaying exponential

Impulse responses for first-order systems

Real Exponentials

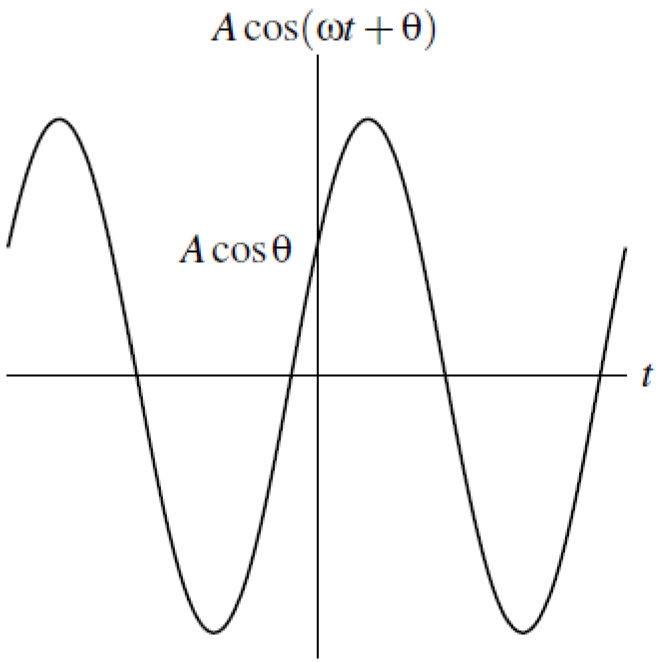
- Gerçek bir üstel, karmaşık bir üstel x (t) = Ae ^ (λt) özel durumudur, burada A ve λ, gerçek sayılarla sınırlıdır.
- Gerçek bir üstel, aşağıda gösterildiği gibi λ değerine bağlı olarak üç farklı davranış tarzından birini gösterebilir.
- λ> 0 ise, x (t), t arttıkça üssel olarak artar (yani, artan bir üstel).
- λ <0 ise, x (t), t arttıkça üssel olarak azalır (yani, azalan bir üstel).
- Eğer λ = 0 ise, x (t) basitçe A sabitine eşittir.



$\lambda > 0$ $\lambda = 0$ $\lambda < 0$

Real Sinusoids

- A (CT) real sinusoid is a function of the form $x(t) = A\cos(\omega t + \theta)$, where A, ω, and θ are real constants.
- Such a function is periodic with fundamental period $T = 2\pi/|\omega|$ and fundamental frequency $|\omega|$.
- A real sinusoid has a plot resembling that shown below.



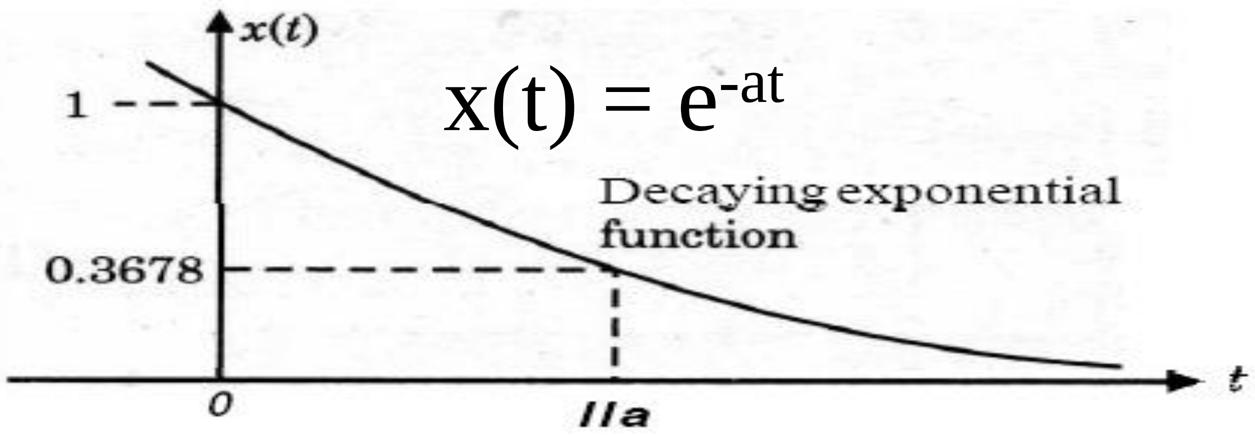
Complex Exponentials

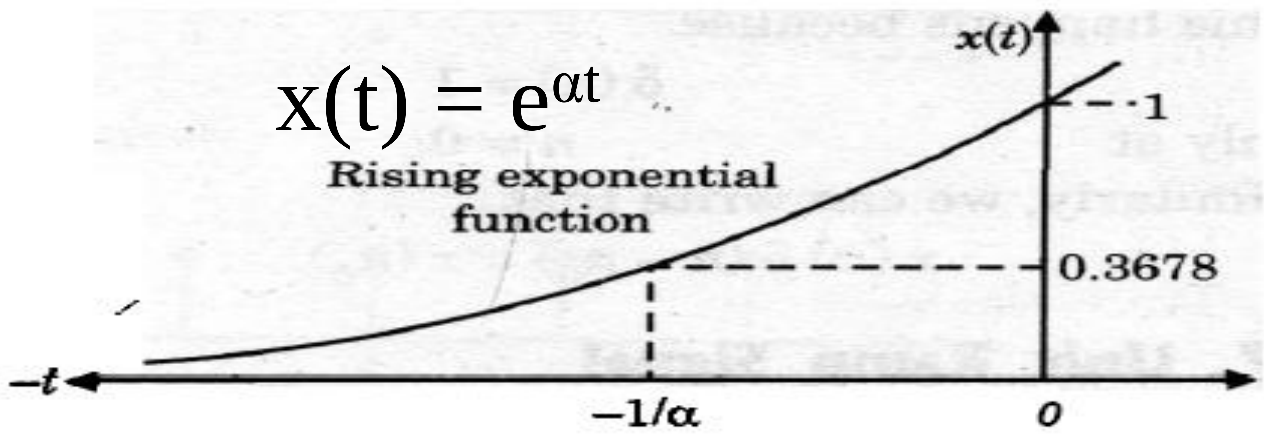
- Bir (CT) karmaşık üstel, A ve λ'nın karmaşık sabitler olduğu x (t) = Ae ^ (λt) formundaki bir fonksiyondur.
- Karmaşık bir üstel, A ve λ parametrelerinin değerlerine bağlı olarak bir dizi farklı davranış tarzından birini gösterebilir.
- Örneğin, özel durumlar olarak, karmaşık üsteller gerçek üstelleri ve karmaşık sinüzoidleri içerir.

Complex Sinusoids

- A complex sinusoid is a special case of a complex exponential $x(t) = Ae^{\lambda t}$, where A is complex and λ is purely imaginary (i.e., $Re\{\lambda\} = 0$).
- That is, a (CT) complex sinusoid is a function of the form $x(t) = Ae^{j\omega t}$, where A is complex and ω is real.
- By expressing A in polar form as $A = |A|e^{j\theta}$ (where θ is real) and using
- Euler's relation, we can rewrite x(t) as $x(t) = |A|\cos(\omega t + \theta) + j\,|A|\sin(\omega t + \theta)$
- $Re\{x(t)\} = |A|\cos(\omega t + \theta)$, $Im\{x(t)\} = |A|\sin(\omega t + \theta)$
- Thus, $Re\{x\}$ and $Im\{x\}$ are the same except for a time shift.
- Also, x is periodic with fundamental period $T = 2\pi/|\omega|$ and fundamental frequency $|\omega|$.

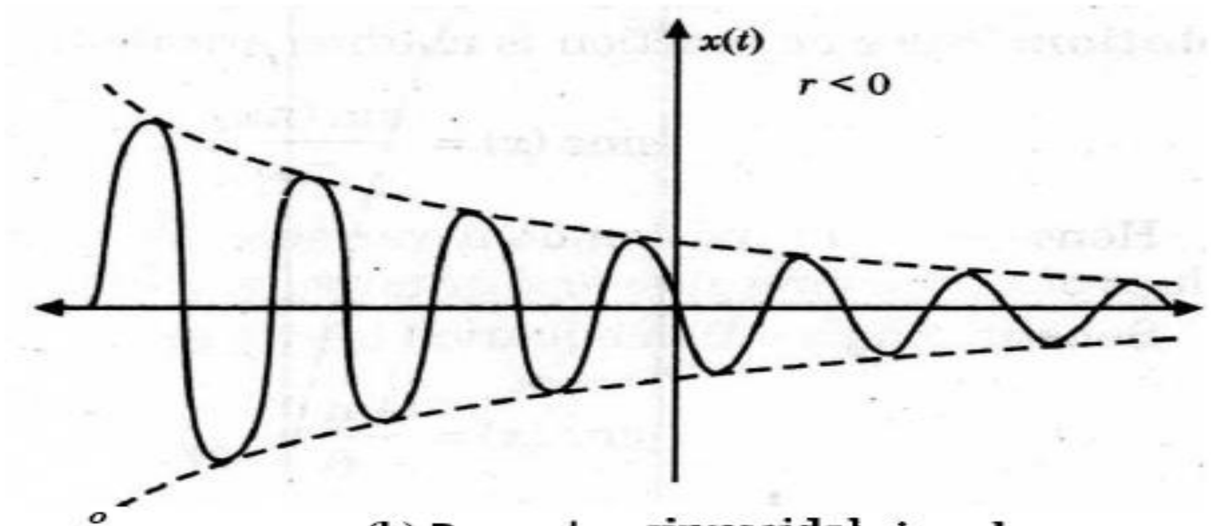
Real Exponential Signals and damped (Sönümlü) Sinusoidal



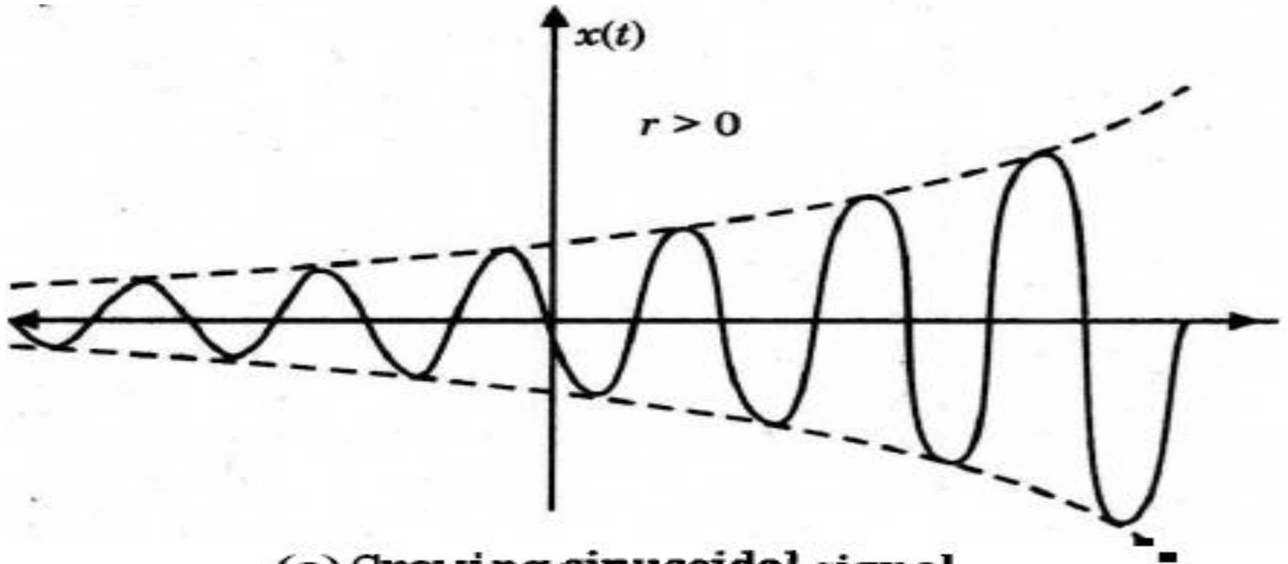


A discrete time exponential signal is expressed as

$x(n) = a^n$



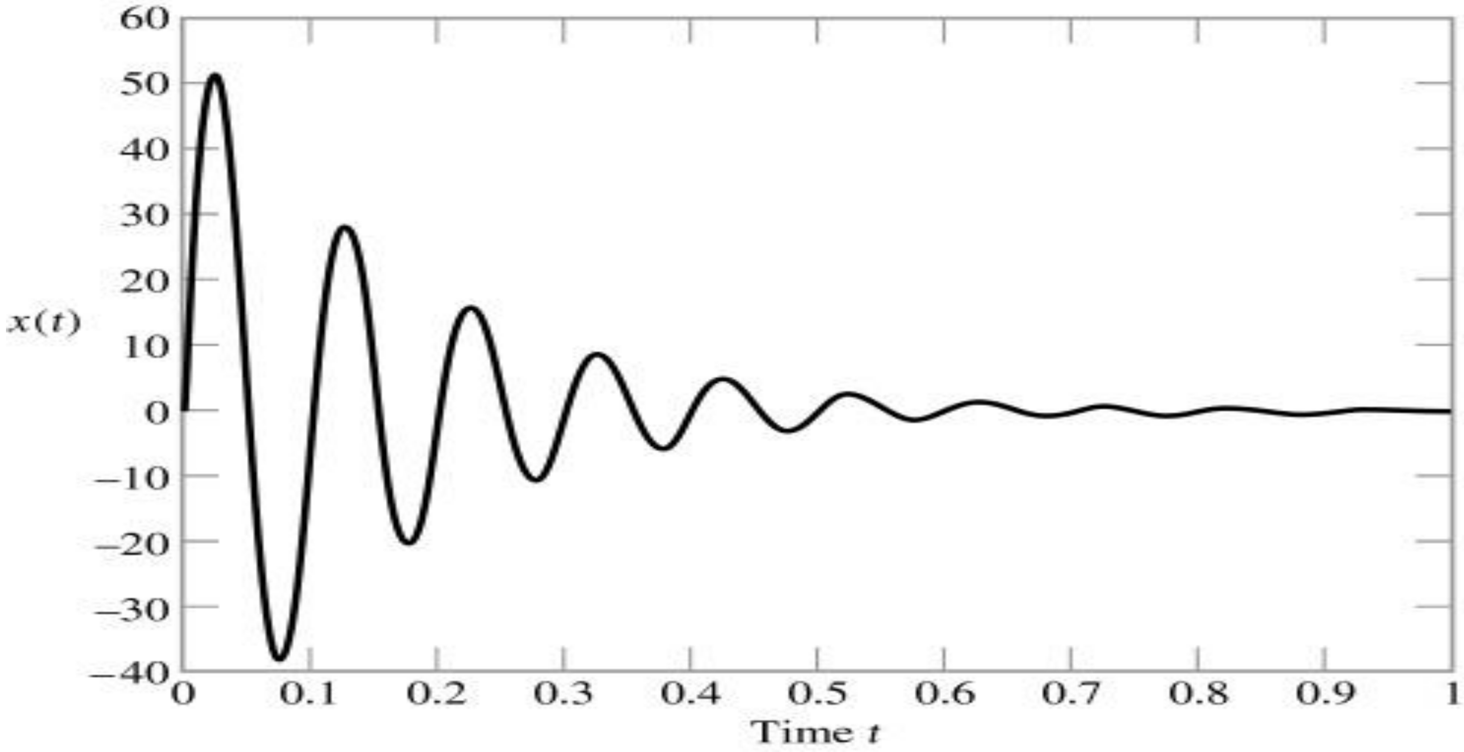
(b) Decaying sinusoidal signal



(a) Growing sinusoidal signal

$$e^{rt} \cos(\omega_0 t + \theta)$$

Damped (Sönümlü) Sinusoidal



$$x(t) = Ae^{-\alpha t}\sin(\omega t + \phi), \quad \alpha > 0$$

Standard Curves

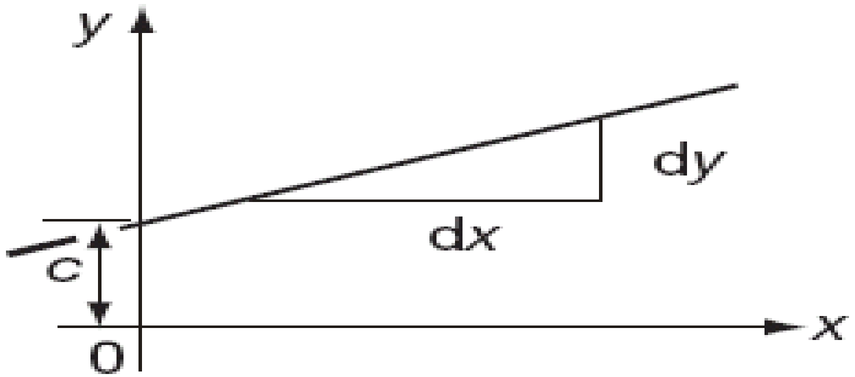
Standard curves

- *Straight line*
- *Second-degree curves*
- *Third-degree curves*
- *Circle*
- *Ellipse*
- *Hyperbola*
- *Logarithmic curves*
- *Exponential curves*
- *Hyperbolic curves*
- *Trigonometrical curves*

Standard curves

Straight line

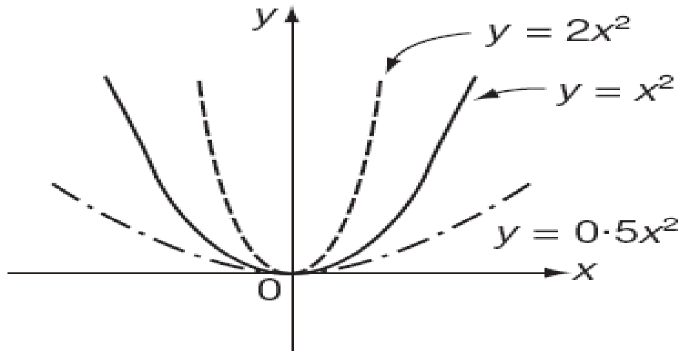
The equation of a straight line is a first-degree relationship and can always be expressed in the form: $y = mx + c$
where m = dy/dx is the gradient of the line and c is the y value where the line crosses the y-axis – the *vertical intercept*.



Standard curves

Second-degree curves

The simplest second-degree curve is expressed by: $y = x^2$
Its graph is a parabola, symmetrical about The y-axis and existing only for $y \geq 0$.
$y = ax^2$ gives a thinner parabola if $a > 1$ and a flatter parabola if $0 < a < 1$. The general second-degree curve is: $y = ax^2 + bx + c$
where a, b and c determine the position, 'width' and orientation of the parabola.



Standard curves

Second-degree curves (change of vertex)

If the parabola: $y = x^2$

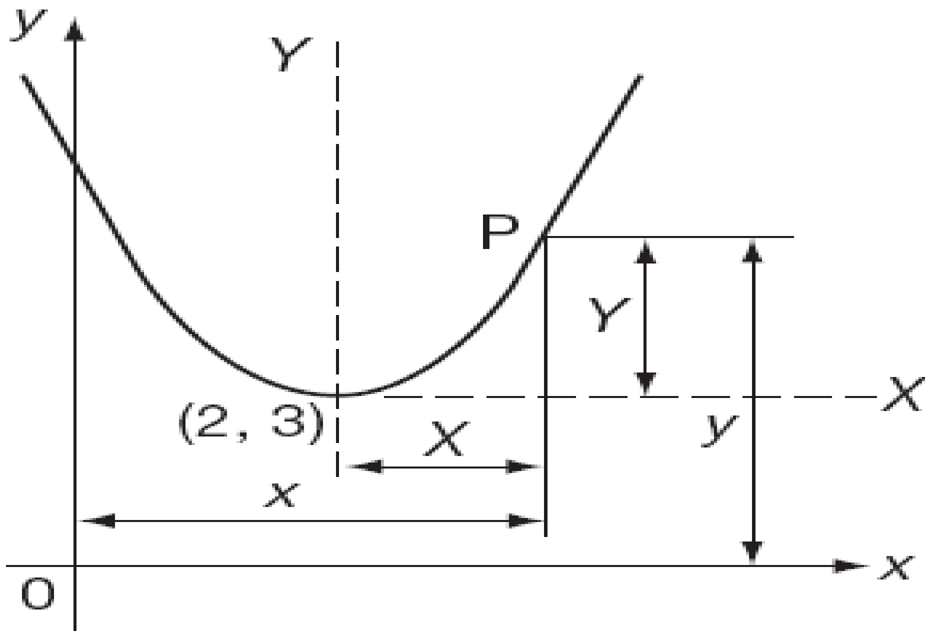
is moved parallel to itself to a vertex position (2, 3), for example, its equation relative to the new axes is

$$Y = X^2$$

where $Y = y - 3$ and $X = x - 2$.

Relative to the original axes this gives

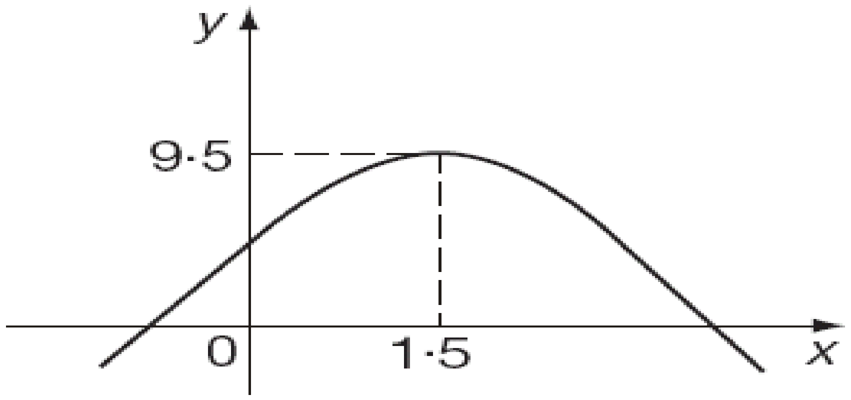
$$y = x^2 - 4x + 7$$



Standard curves

Second-degree curves

If: $y = ax^2 + \text{bx} + c$
and $a < 0$ then the parabola is inverted.
For example: $y = -2x^2 + 6x + 5$



y'=-4x+6
y''=-4

- Birinci türev ifadesinde, x=0 için y'=6, y'>0 olduğundan artan durumdadır; x=2 için y'=-2, y' <0 olduğundan azalan durumdadır.
- İkinci türev değeri, y''<0 olduğundan y fonksiyonun maksimumu vardır. Maksimum noktası, y'=0 alınarak x=1.5 bulunur. Maksimum değeri, ymax=9.5 olur.

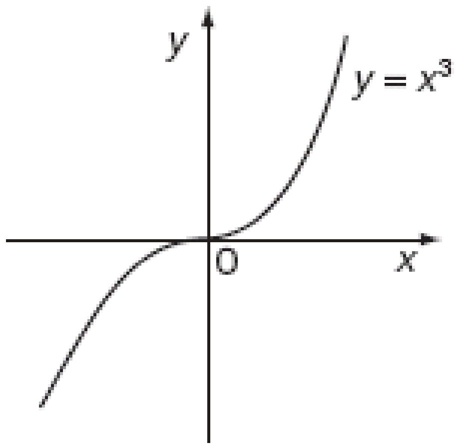
Standard curves

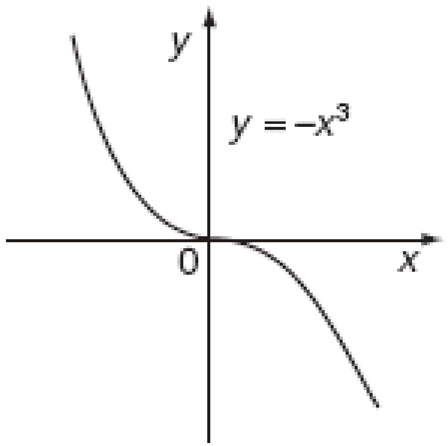
Third-degree curves

The basic third-degree curve is: $y = x^3$

which passes through the origin.

The curve: $y = x^3$ is the reflection in the vertical axis.

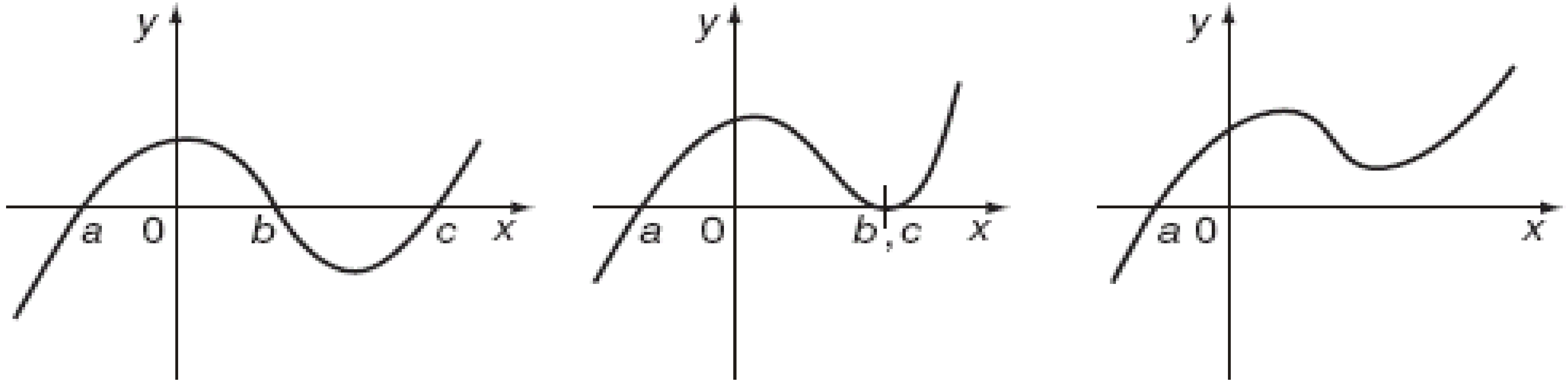




Standard curves

Third-degree curves

The general third-degree curve is: $y = px^3 + qx^2 + rx + s$
Which cuts the *x*-axis at least once.

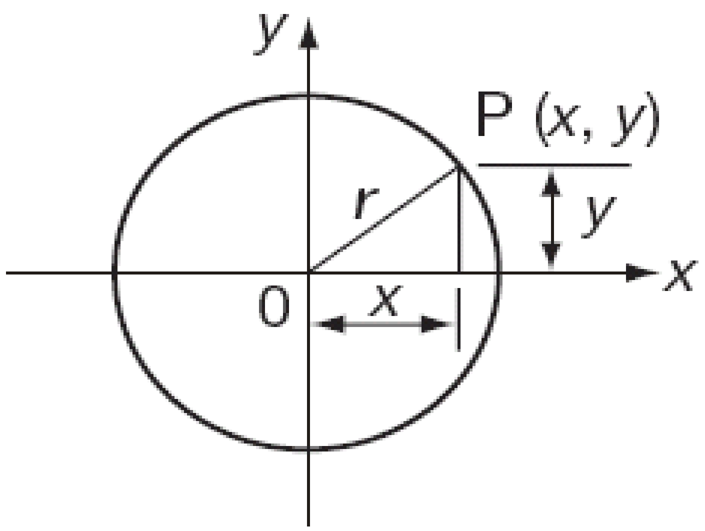


Standard curves

Circle

The simplest case of the circle is with centre at the origin and radius r.

The equation is then $x^2 + y^2 = r^2$



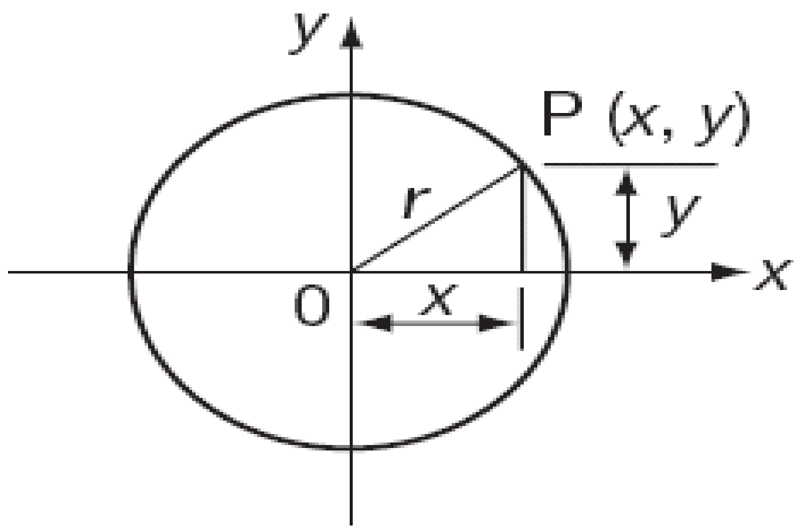
Standard curves

Circle

Moving the centre to (h, k) gives: $X^2 + Y^2 = r^2$

where: $X = x - h$

$Y = y - k$



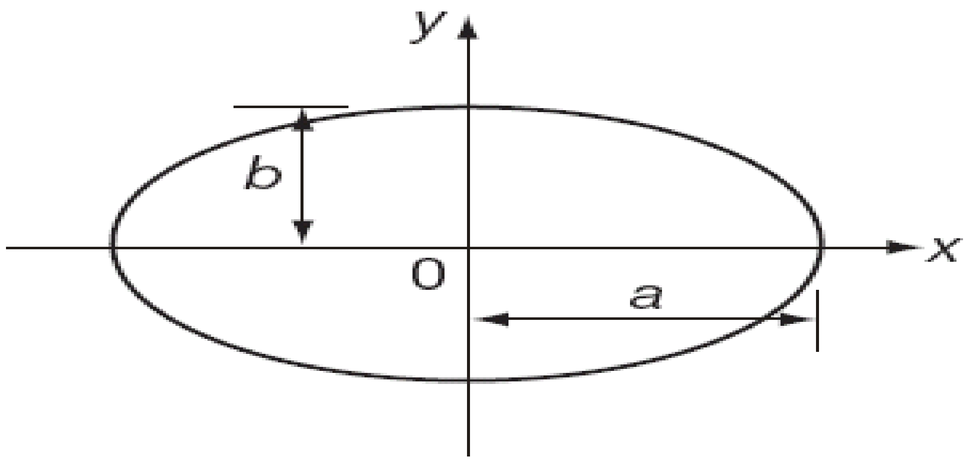
The general equation of a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

centre $(-g, -f)$ radius $\sqrt{g^2 + f^2 - c}$

Standard curves

Ellipse

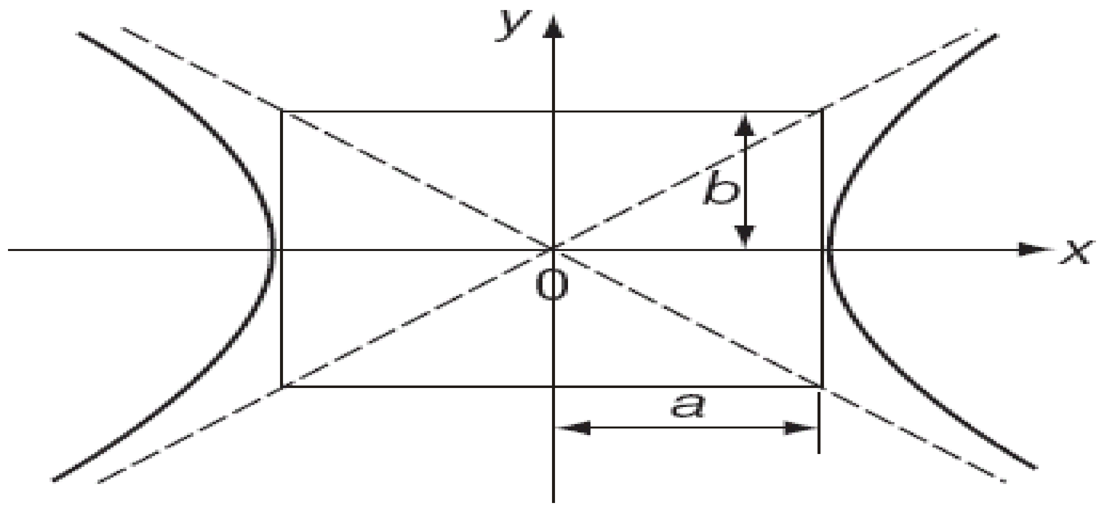


The equation of an ellipse is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If $a > b$ then a is called the semi-major axis and b is called the semi-minor axis.

Standard curves

Hyperbola



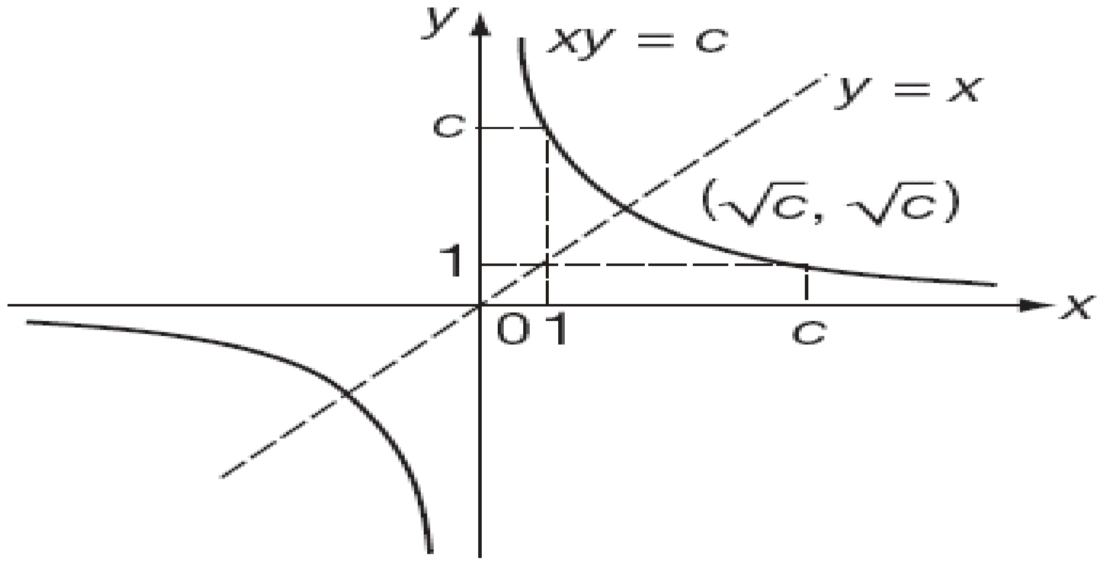
The equation of an hyperbola is: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

When $y = 0$, $x = \pm a$ and when $x = 0$, $y^2 = -b^2$ and the curve does not cross the y-axis.

Note: The two opposite arms of the hyperbola gradually approach two straight lines (*asymptotes*).

Standard curves

Rectangular hyperbola



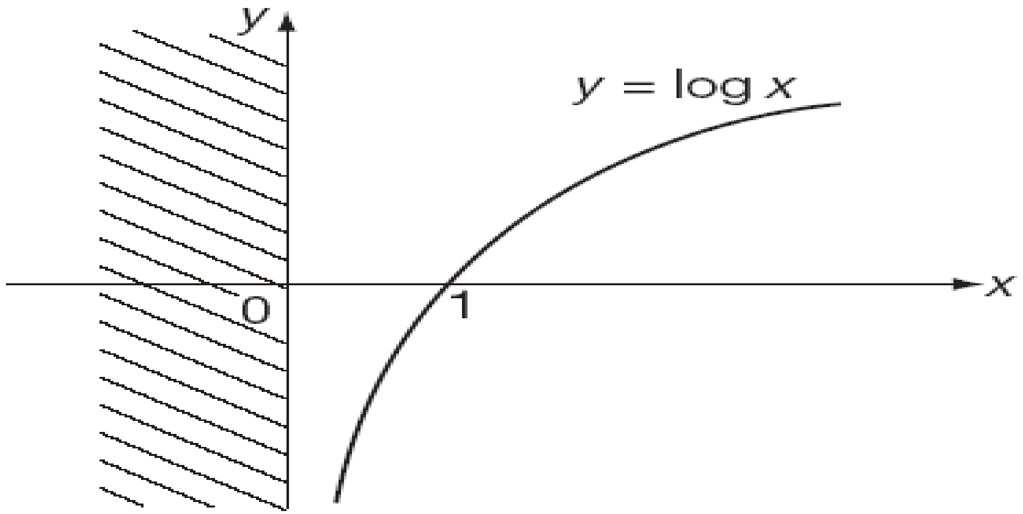
If the asymptotes are at right angles to each other, the curve is a *rectangular hyperbola*.

If the curve is rotated through 45^o so that the asymptotes coincide with the coordinate axes the equation is then:

$$xy = c \quad \text{that is} \quad y = \frac{c}{x}$$

Standard curves

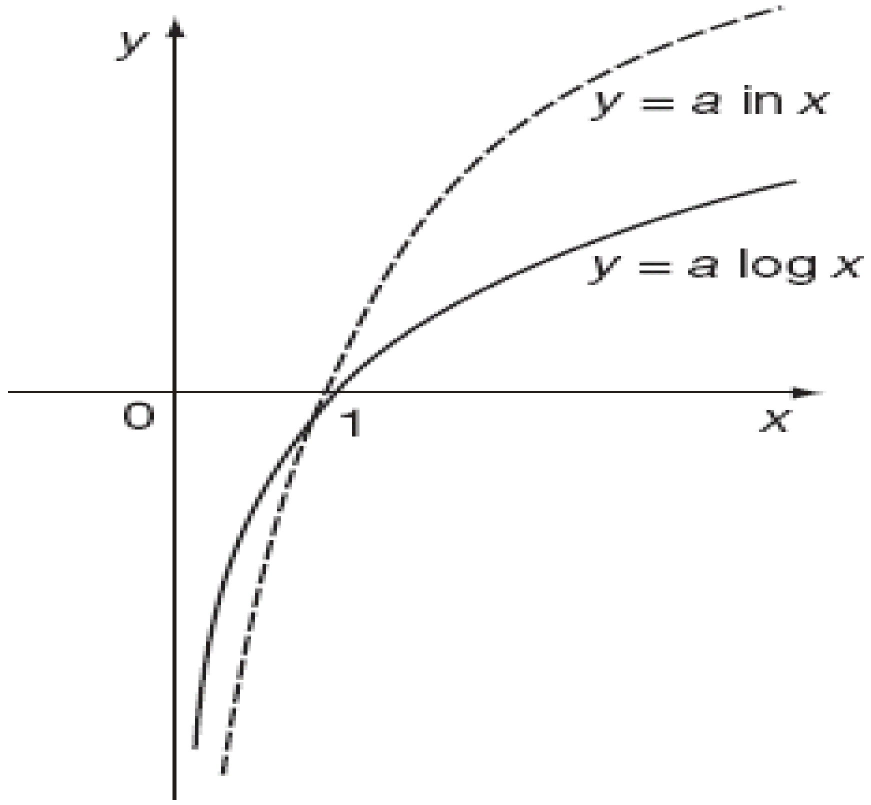
Logarithmic curves



If $y = \log x$, then when: $\mathrm{x} = 1$ $then\ y = log1 = 0$
so the curve crosses the x-axis at $x = 1$
Also, log x does not exist for real $x < 0$.

Standard curves

Logarithmic curves



The graph of $y = \ln x$ also has the same shape and crosses the x-axis at $x = 1$.
The graphs of $y = a\log x$ and $y = a\ln x$ are similar but with all ordinates multiplied by the constant factor a.

Standard curves

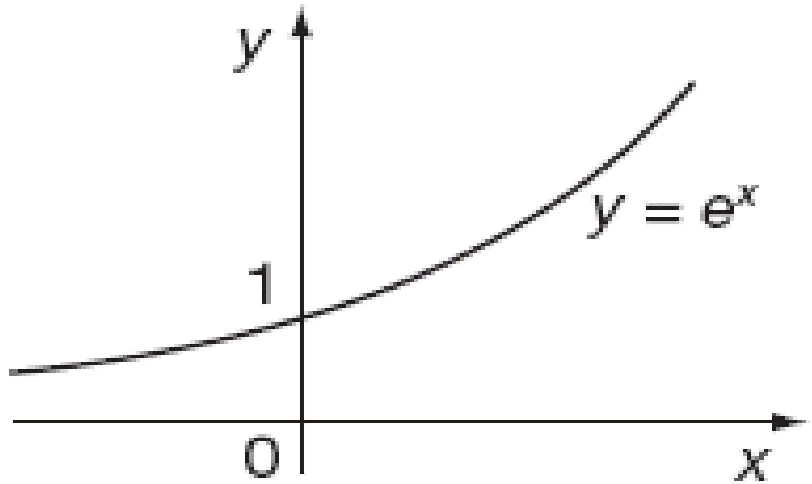
Exponential curves

The curve $y = e^x$ crosses the y-axis at $x = 0$.

As $x \to \infty$ so $y \to \infty$

as $x \to -\infty$ so $y \to 0$

Sometimes called the *growth curve*.



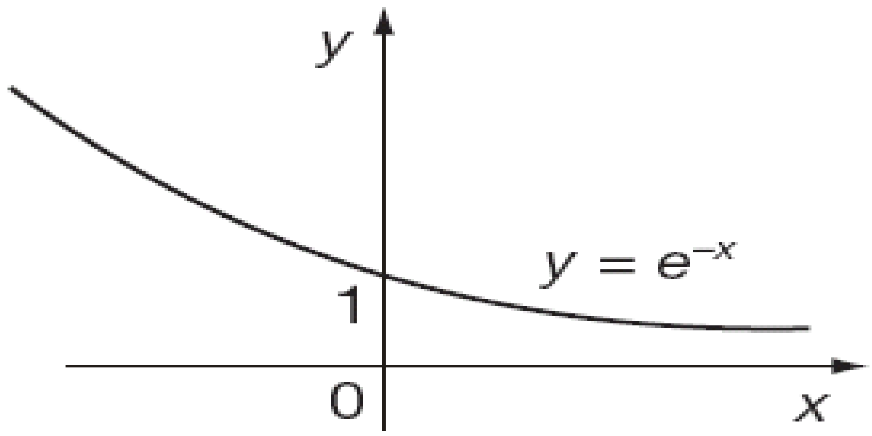
Standard curves

Exponential curves

The curve $y = e^{-x}$ crosses the y-axis at $y = 1$.

As $x \rightarrow \infty$ so $y \rightarrow 0$

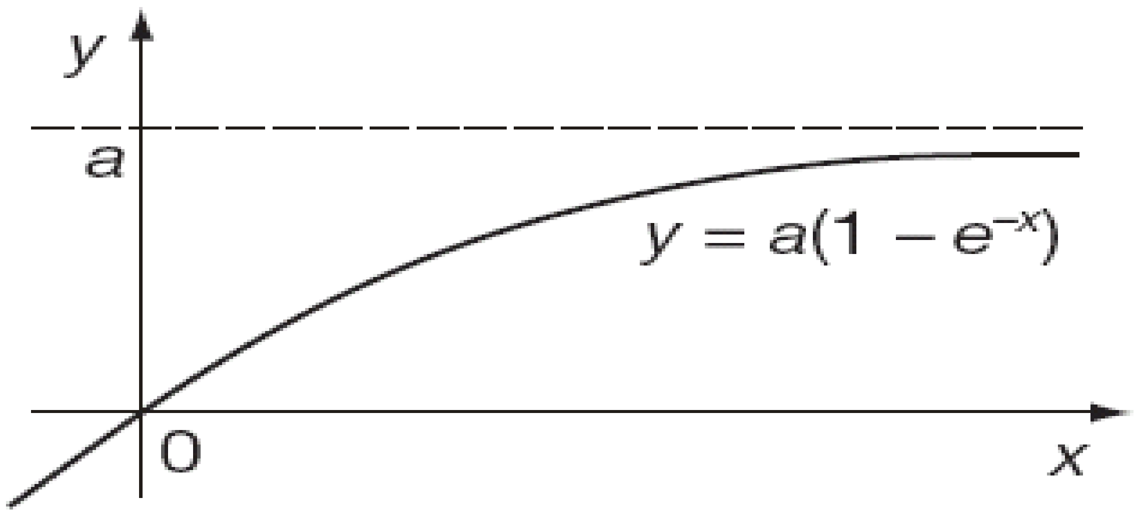
as $x \rightarrow -\infty$ so $y \rightarrow \infty$



Sometimes called the *decay curve*.

Standard curves

Exponential curves



The curve: $y = a\left(1 - e^{-x}\right)$

passes through the origin and tends to the asymptote $y = a$ as $x \to \infty$.

Standard curves

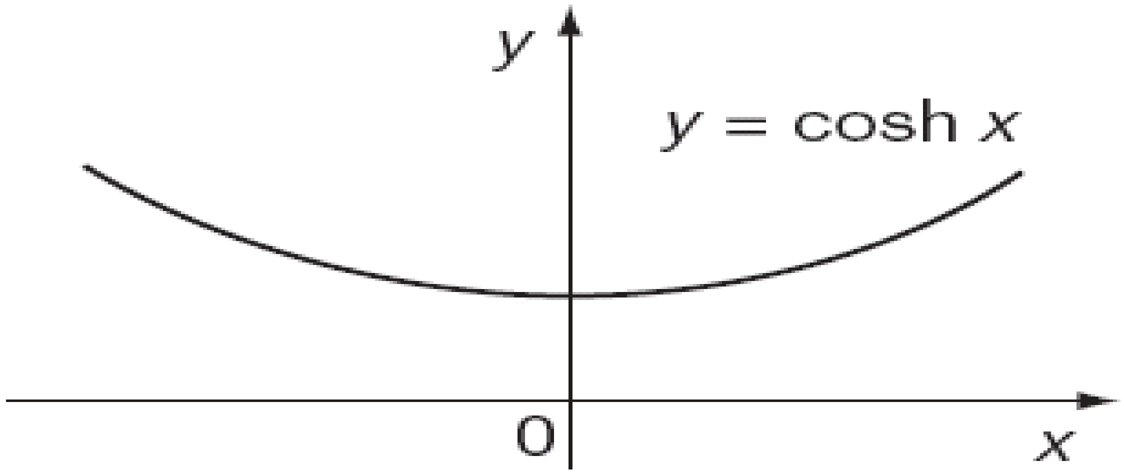
Hyperbolic curves

The combination of the curves for:

$$y = e^{x} \quad \text{and} \quad y = e^{-x}$$

gives the hyperbolic cosine curve:

$$y = \cosh x = \frac{e^{x} + e^{-x}}{2}$$



Standard curves

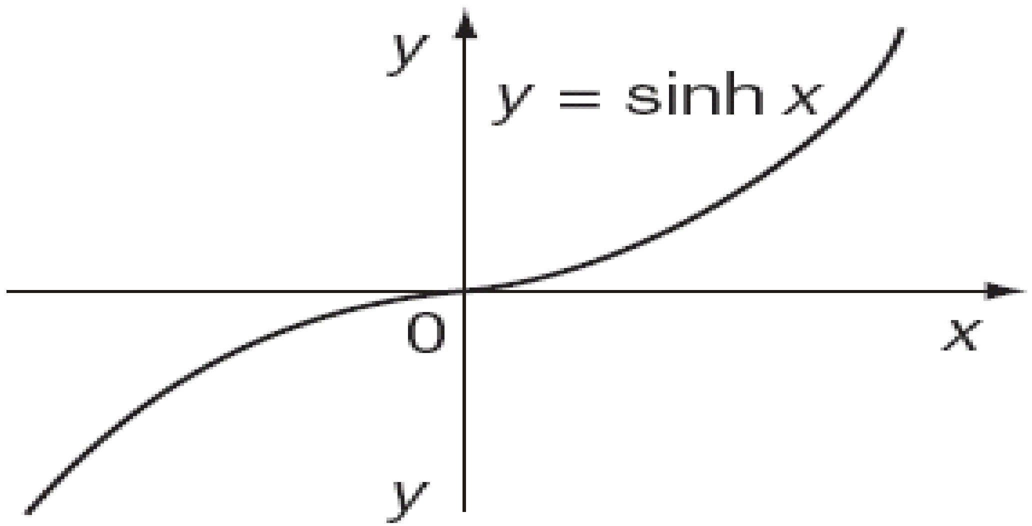
Hyperbolic curves

Another combination of the curves for:

$$y = e^{x} \quad \text{and} \quad y = e^{-x}$$

gives the hyperbolic sine curve:

$$y = \sinh x = \frac{e^{x} - e^{-x}}{2}$$



Standard curves

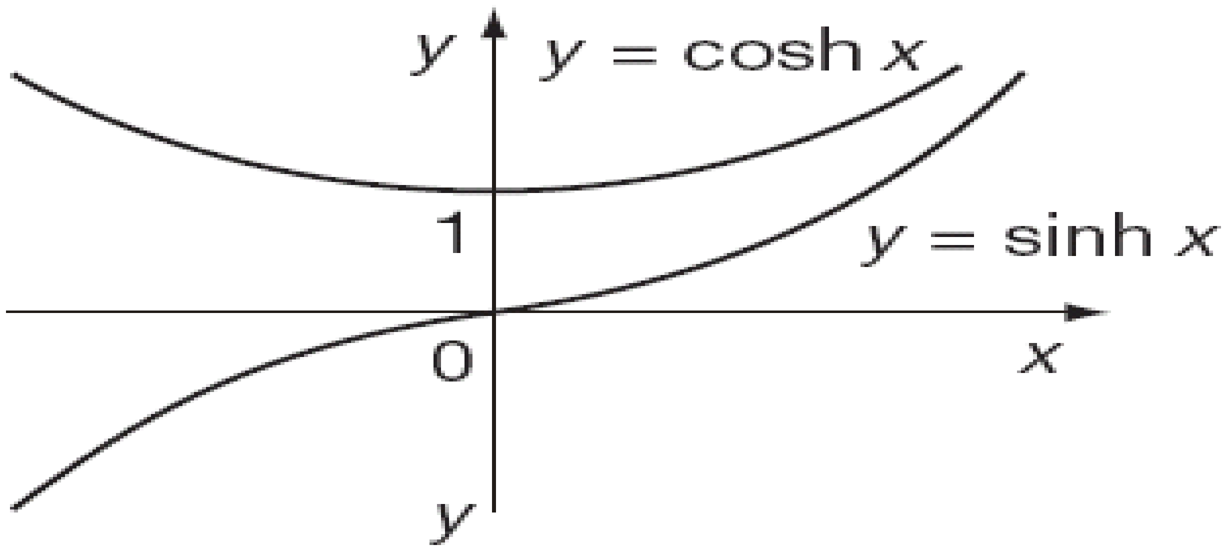
Hyperbolic curves

Plotting these last two curves together shows that:

$$y = \sinh x$$

is always outside:

$$y = \cosh x$$



Standard curves

Trigonometrical curves

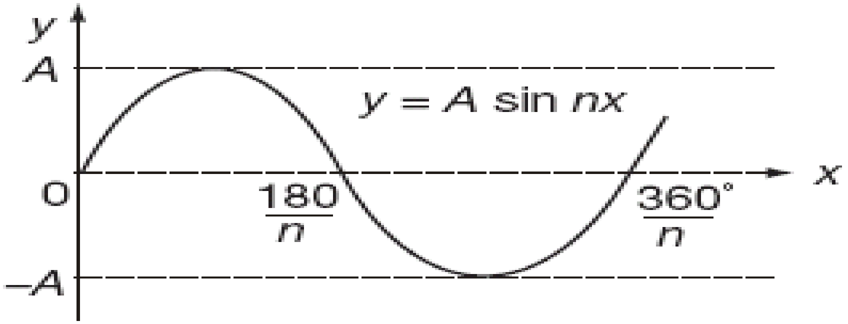
The sine curve is given as:

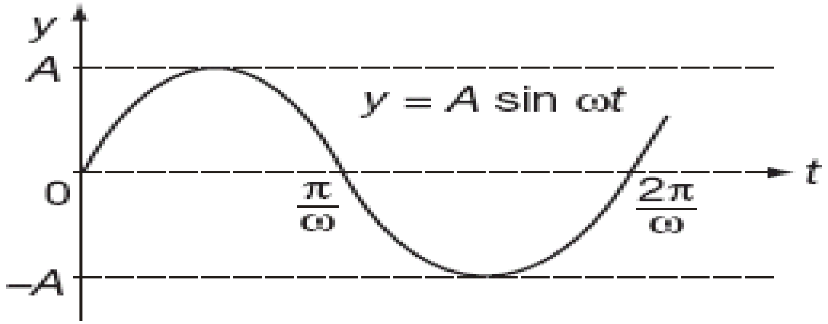
(a) $y = A \sin nx$ where

$$\text{Period} = \frac{360^\circ}{n}, \text{ amplitude} = A$$

(b) $y = A \sin \omega t$ where

$$\text{Period} = \frac{2\pi}{\omega}, \text{ amplitude} = A$$





Asymptotes

Determination of an asymptote

An asymptote to a curve is a line to which the curve approaches as the distance from the origin increases. To find the asymptote to: y=f(x)

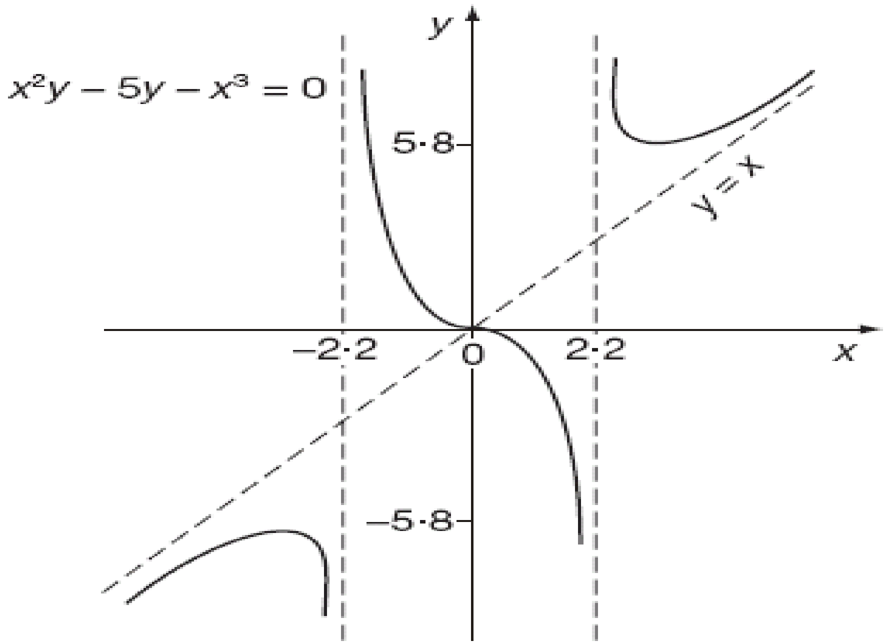
(a)Substitute $y = mx + c$ in the given equation and simplify

(b)Equate to zero the coefficients of the two highest powers of x

(c)Determine the values of m and c to find the equation of the asymptote.

Asymptotes

Determination of an asymptote



For example, to find the asymptote to the curve: $x^2y - 5y - x^3 =$

Substitute $y = mx + c$ into the equation to obtain: $(m-1)x^3 + cx^2 - 5mx - 5c = 0$

Equate the coefficients of x^3 and x^2 to zero to obtain: $m = 1$ and $c = 0$

Giving the asymptote: $y = x$

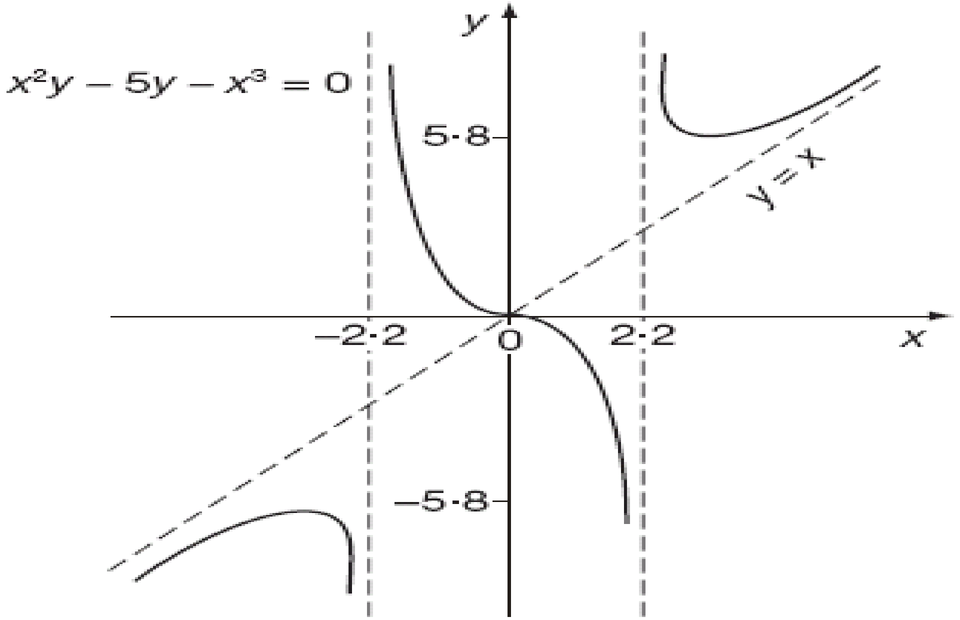
Asymptotes

Asymptotes parallel to the x- and y-axes

For the curve $y = f(x)$, the asymptotes parallel to the y-axis can be found by equating the coefficient of the highest power of y to zero.

Therefore for: $x^2y - 5y - x^3 = 0$

The asymptotes are given by: $x^2 - 5 = 0$



That is: $x = \pm\sqrt{5}$
$\cong \pm 2.2$

Asymptotes

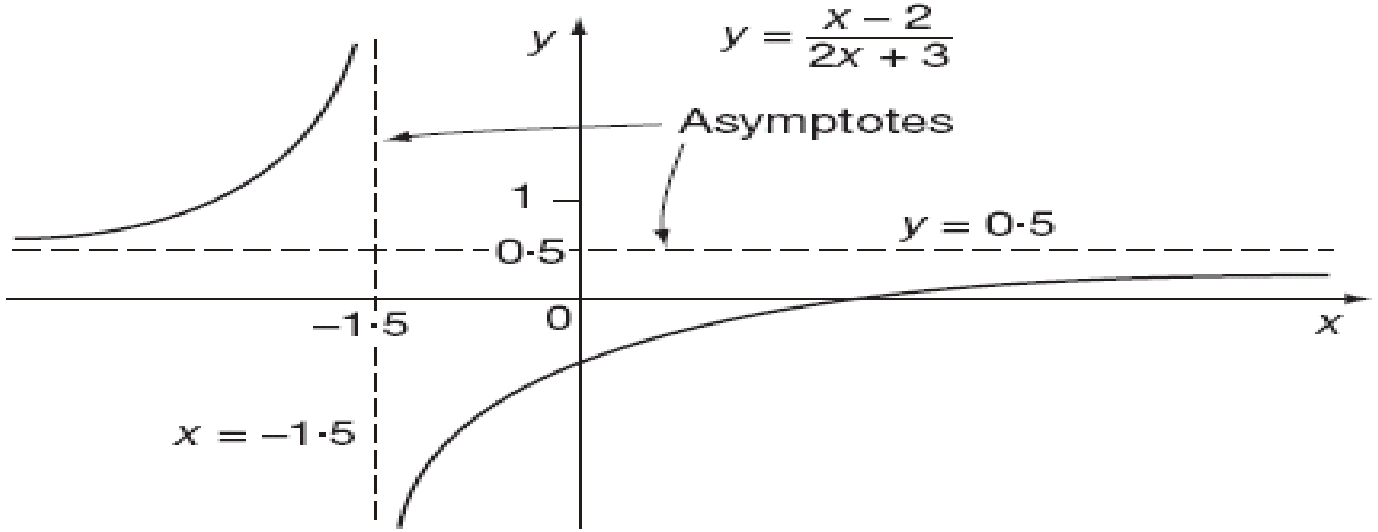
Asymptotes parallel to the x- and y-axes

For the curve $y = f(x)$, the asymptotes parallel to the x-axis can be found by equating the coefficient of the highest power of x to zero.

Therefore for: $(2x+3)y - x + 2 = 0$

The asymptote is given by: $2y - 1 = 0$

That is: $y = 0.5$



Systematic curve sketching, given the equation of the curve

Symmetry

Intersection with the axes

Change of origin

Asymptotes

Large and small values of x *and* y

Stationary points

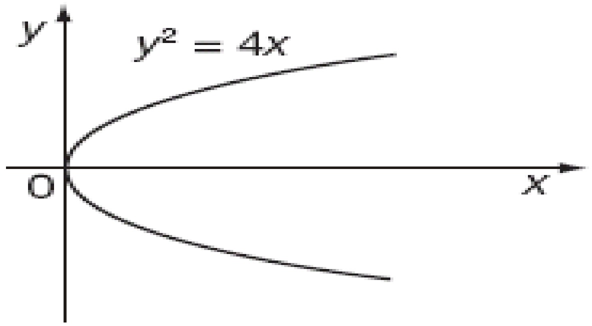
Limitations

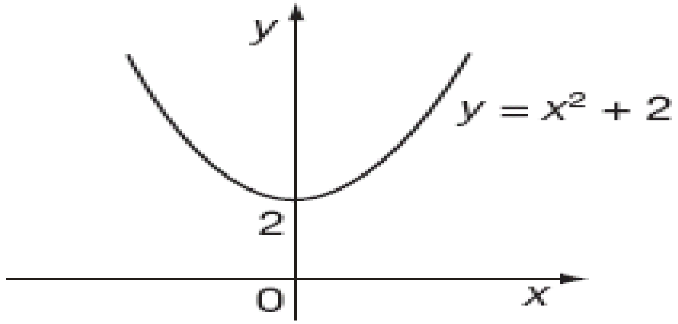
Systematic curve sketching, given the equation of the curve

Symmetry

Inspect the equation for symmetry:

(a)If only even powers of *y* occur, the curve is symmetrical about the *x*-axis
(b)If only even powers of *x* occur, the curve is symmetrical about the *y*-axis





(c)If only even powers of *x* and *y* occur, the curve is symmetrical about both axes

Systematic curve sketching, given the equation of the curve

Intersection with the axes

Points at which the curve crosses the x- and y-axes:

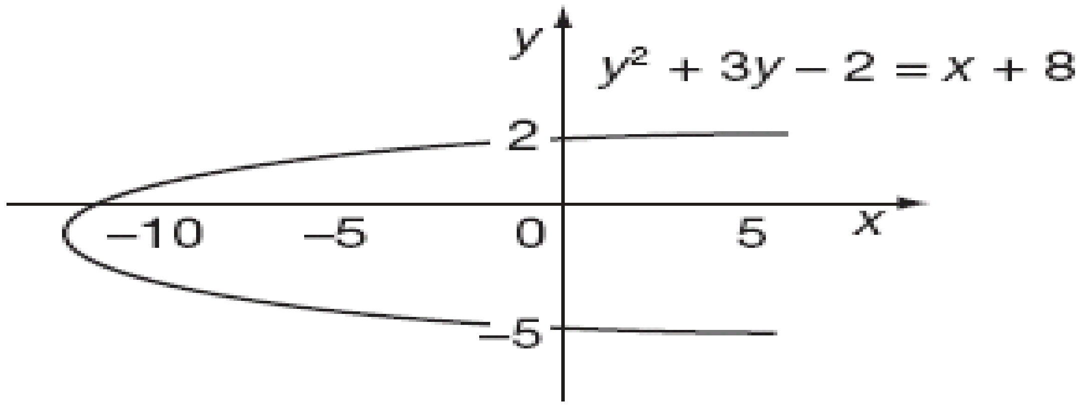
Crosses the x-axis: Put $y = 0$ and solve for x
Crosses the y-axis: Put $x = 0$ and solve for y

For example, the curve

$$y^2 + 3y - 2 = x + 8$$

Crosses the x-axis at $x = -10$

Crosses the y-axis at $y = 2$ and -5

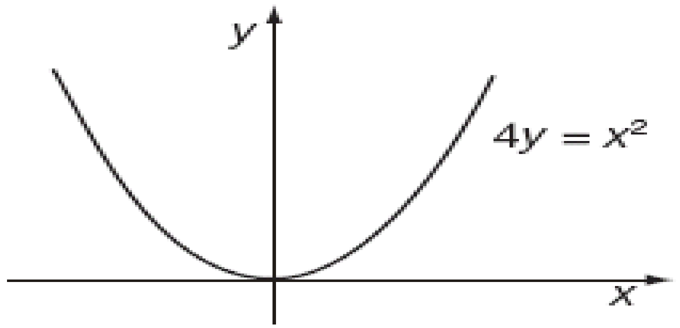


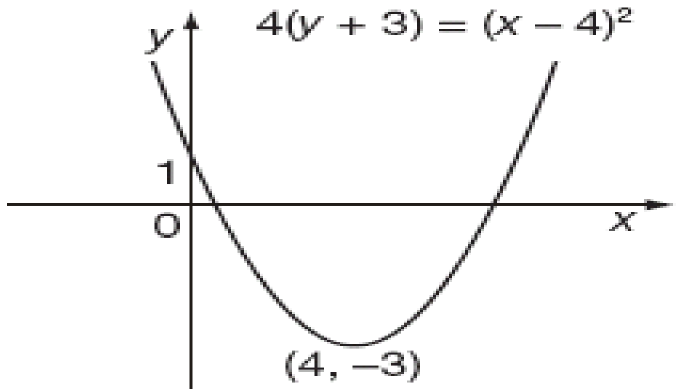
Systematic curve sketching, given the equation of the curve

Change of origin

Look for a possible change of origin to simplify the equation. For example, if, for the curve $4(y+3)=(x-4)^2$

The origin is changed by putting $Y = y + 3$ and $X = x - 4$, the equation becomes that of a parabola symmetrical about the Y axis: $4Y = X^2$





Systematic curve sketching, given the equation of the curve

Asymptotes

The asymptotes parallel with the coordinate axes are found by:

(a)For the curve $y = f(x)$, the asymptotes parallel to the x-axis can be found by equating the coefficient of the highest power of x to zero.

(b)For the curve $y = f(x)$, the asymptotes parallel to the y-axis can be found by equating the coefficient of the highest power of y to zero.

(c)General asymptotes are found by substituting $y = mx + c$ in the given equation, simplifying and equating to zero the coefficients of the two highest powers of x to find the values of m and c.

Systematic curve sketching, given the equation of the curve

Large and small values of x and y

If x or y is small, higher powers of x or y become negligible and hence only lower powers of x or y appearing in the equation provide an approximate simpler form

Systematic curve sketching, given the equation of the curve

Stationary points

Stationary points exists where: $\frac{dy}{dx} = 0$

If further:

$\frac{d^2y}{dx^2} < 0$ the stationary point is a maximum

$\frac{d^2y}{dx^2} > 0$ the stationary point is a minimum

$\frac{d^2y}{dx^2} = 0$ with a change in sign through the stationary point

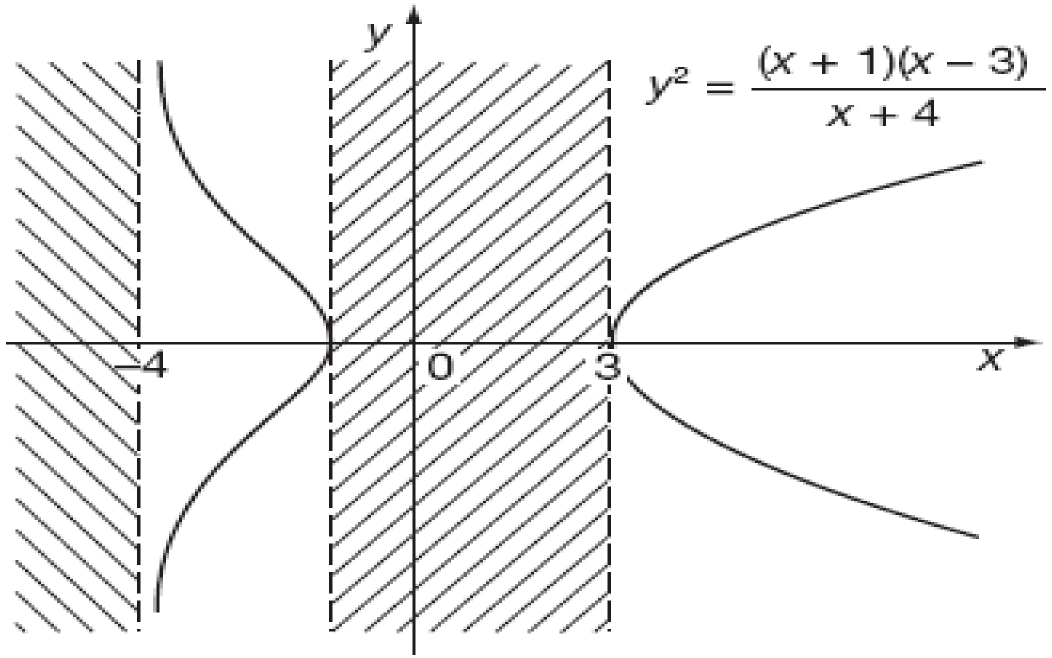
then the point is a point of inflexion

Systematic curve sketching, given the equation of the curve

Limitations

Restrictions on the possible range of values that *x* or *y* may have. For example:

$$y^2 = \frac{(x+1)(x-3)}{x+4}$$



For $x < -4$ $\quad y^2$ is negative (no real y)

For $-4 < x < -1$ $\quad y^2$ is positive

For $-1 < x < 3$ $\quad y^2$ is negative (no real y)

For $3 < x$ $\quad y^2$ is positive

Curve fitting

Straight-line law

Graphs of the form $y = ax^n$*, where a and n are constants*

Graphs of the form $y = ae^{nx}$

Curve fitting

Straight-line law

If the assumption that the two variables *x* and *y* whose values are taken from experiment are linearly related then their relationship will be expressed algebraically as:

$$y = ax + b$$

where *a* represents the gradient of the straight line and *b* represents the vertical intercept

From a plot of the data, a straight line is drawn through the data as the 'line of best fit'. The values of *a* and *b* are then read off from the graph.

Curve fitting

Graphs of the form $y = ax^n$, where a and n are constants

Taking logarithms of both sides of the equation: $y = ax^n$

yields: $\log y = \log a + n \log x$

If data is collected for the *x* and *y* values then these must be converted to *X* and *Y* values where: $X = \log x$ and $Y = \log y$

So that: $Y = \log a + nX$: a straight line gradient n, vertical intercept $\log a$

Curve fitting

Graphs of the form $y = ae^{nx}$

Taking natural logarithms of both sides of the equation: $y = ae^{nx}$

yields: $\ln y = \ln a + nx$

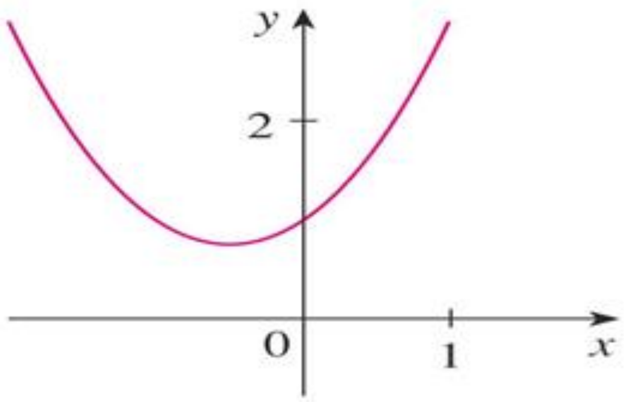
If data is collected for the *x* and *y* values then the *y* values must be converted to *Y* values where: $Y = \ln y$

So that: $Y = \ln a + nx$: a straight line gradient n, vertical intercept $\ln a$

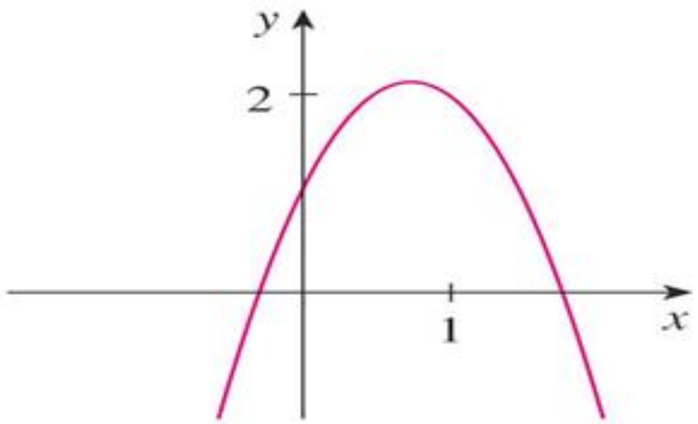
“Fonksiyonlar”

Polynomials

- A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$
- where n is a nonnegative integer and the numbers a0, a1, a2, …, an are constants called the coefficients of the polynomial.
- A polynomial of degree 1 is of the form $P(x) = mx + b$. So, it is a linear function.
- A polynomial of degree 2 is of the form $P(x) = ax^2 + bx + c$. It is called a quadratic function.
- Its graph is always a parabola obtained by shifting the parabola $y = x^2$. The parabola opens upward if $a > 0$ and downward if $a < 0$.



(a) $y = x^2 + x + 1$

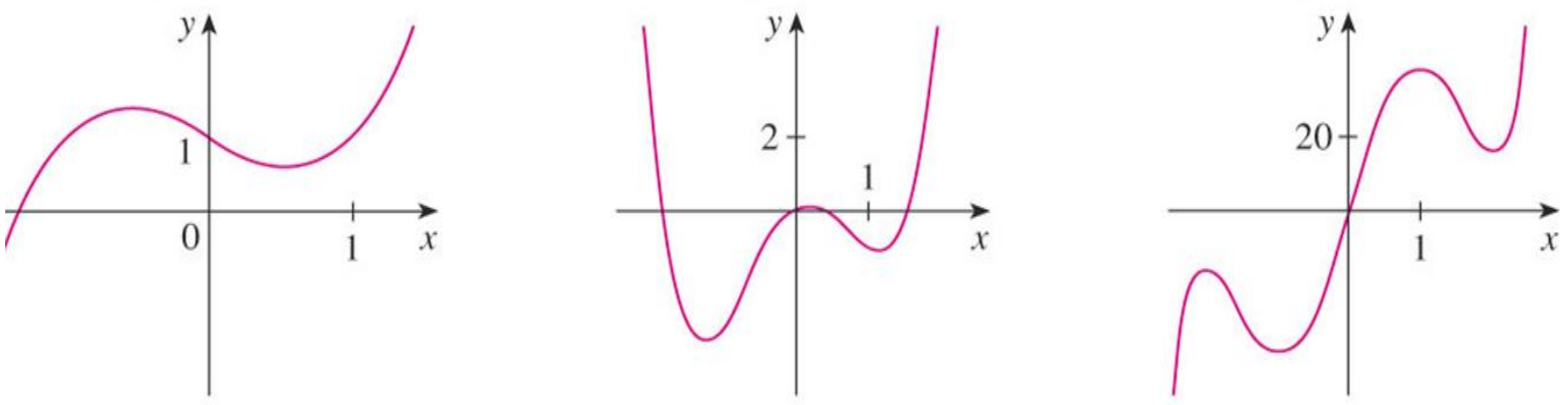


(b) $y = -2x^2 + 3x + 1$

Polynomials are commonly used to model various quantities that occur in the natural and social sciences. A polynomial of degree 3 is of the form

$$P(x) = ax^3 + bx^2 + cx + d \qquad (a \neq 0)$$

It is called a cubic function.



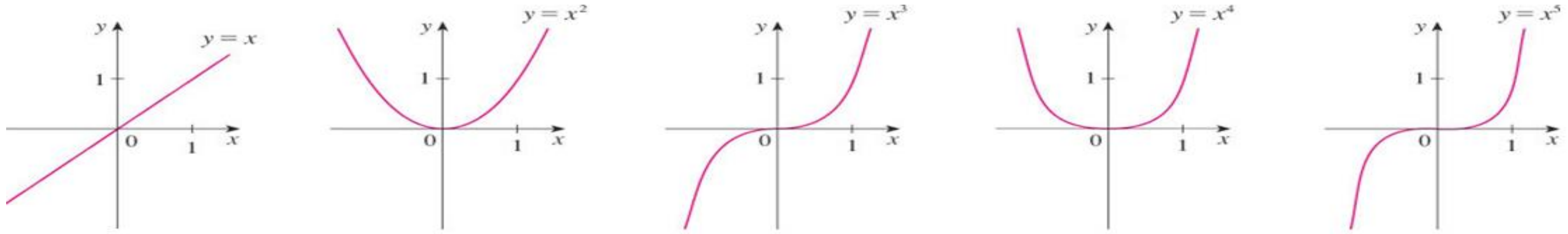
(a) $y = x^3 - x + 1$ (b) $y = x^4 - 3x^2 + x$ (c) $y = 3x^5 - 25x^3 + 60x$

POWER FUNCTIONS

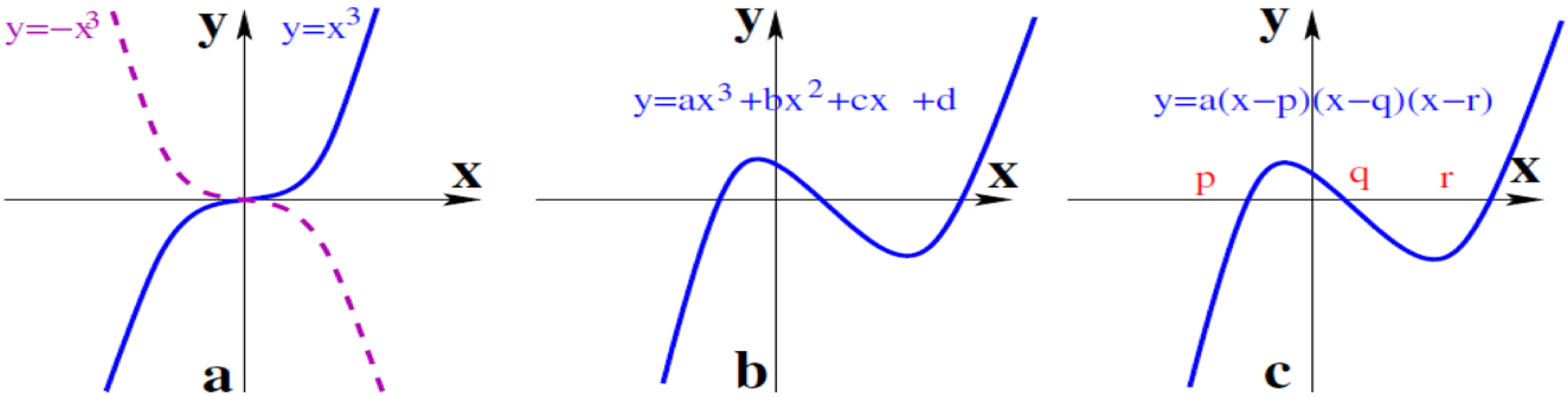
A function of the form $f(x) = x^a$, where a is constant, is called a power function.

$a = n$, where n is a positive integer

- The graphs of $f(x) = x^n$ for n = 1, 2, 3, 4, and 5 are shown.
- These are polynomials with only one term.
- We already know the shape of the graphs of $y = x$ (a line through the origin with slope 1) and $y = x^2$ (a parabola).



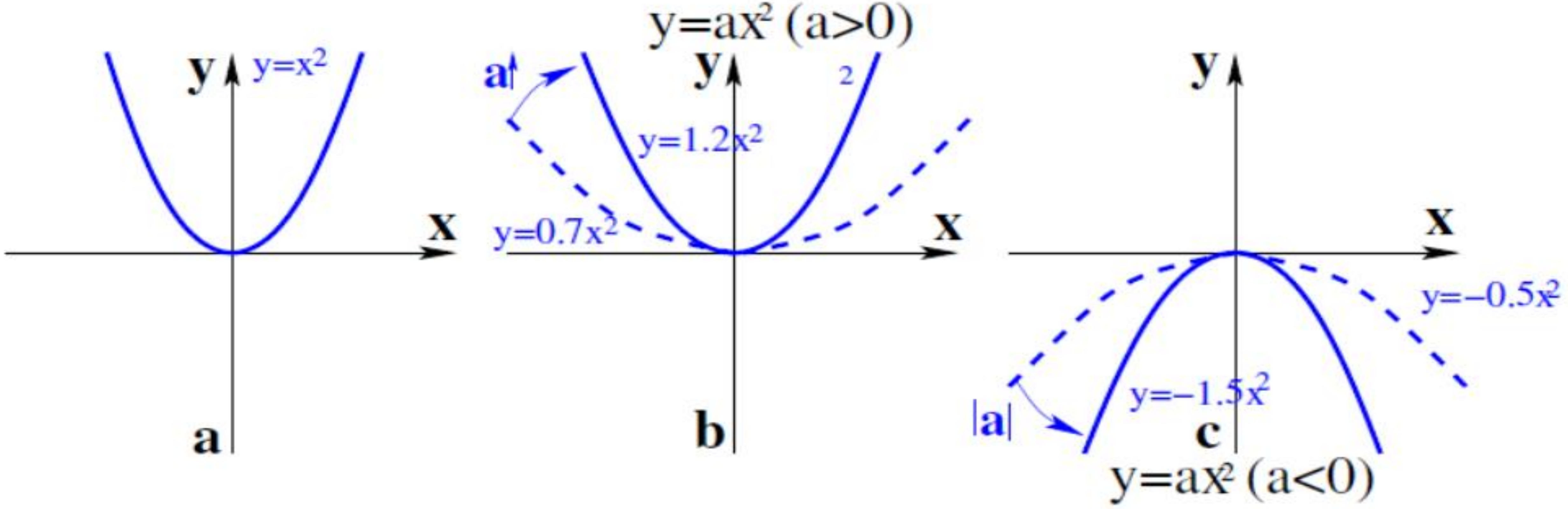
Qubic Function



A general cubic function $y = ax^3 + bx^2 + cx + d$

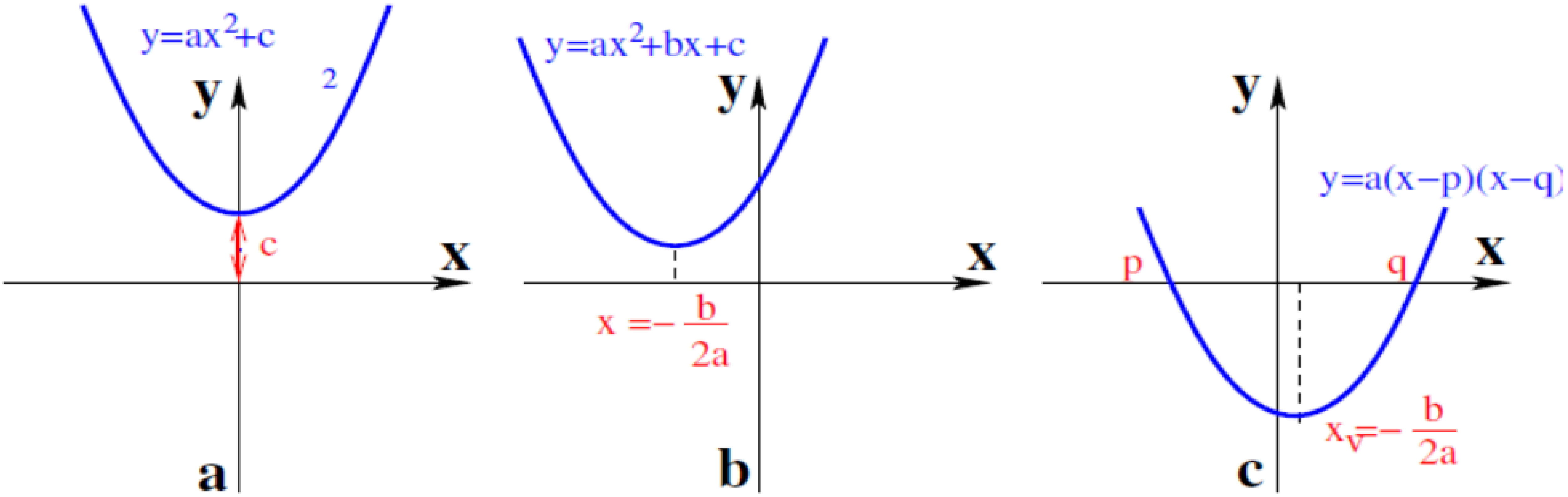
The extrema are points where the derivative of the function is zero, which in this case results in the following quadratic equation: $(ax^3+bx^2+cx+d)' = 3ax^2+2bx+c = 0$.

Parabolik function



Equation $y = ax^2$ produces a parabola, if $a > 0$ the parabola is opened upward (fig.a,b), and if $a < 0$ the parabola is opened downward (fig.c). The larger the absolute value of a is, the steeper is the parabola.

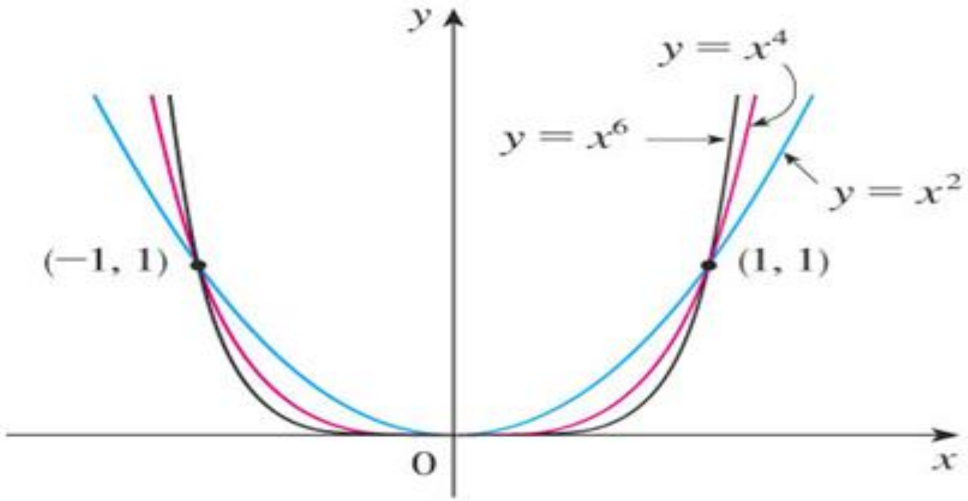
Parabolik fonksiyonlar

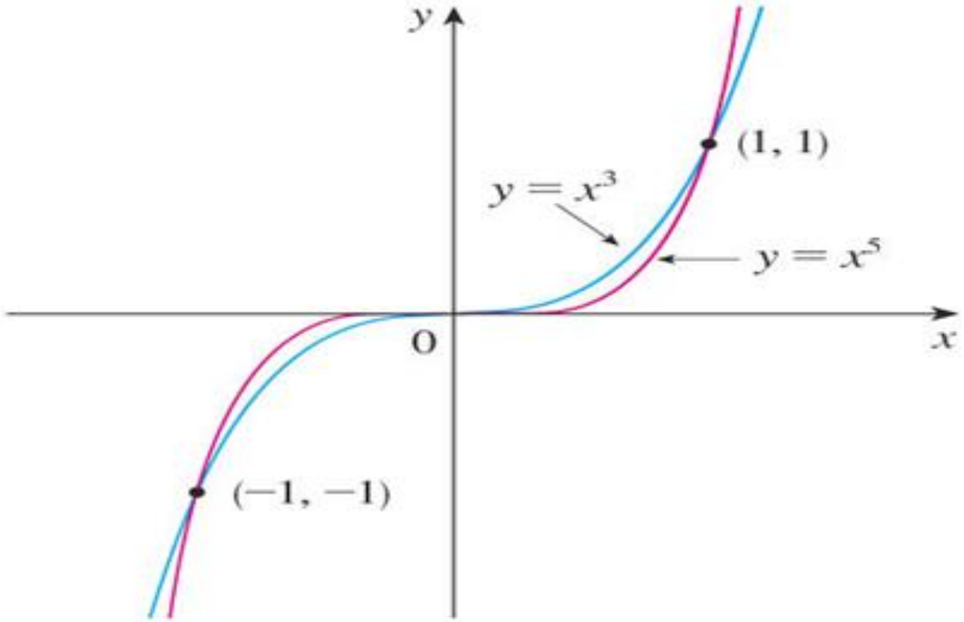


Equation y = ax^2+bx+c also produces a parabola. At this point the derivative of the function to zero (ax^2 +bx+c)' = 2ax+b = 0

CASE

- The general shape of the graph of $f(x) = x^n$ depends on whether n is even or odd.
- If n is even, then $f(x) = x^n$ is an even function, and its graph is similar to the parabola $y = x^2$.
- If n is odd, then $f(x) = x^n$ is an odd function, and its graph is similar to that of $y = x^3$.
- However, notice from the figure that, as n increases, the graph of $y = x^n$ becomes flatter near 0 and steeper when $|x| \geq 1$. If x is small, then x^2 is smaller, x^3 is even smaller, x^4 is smaller still, and so on.

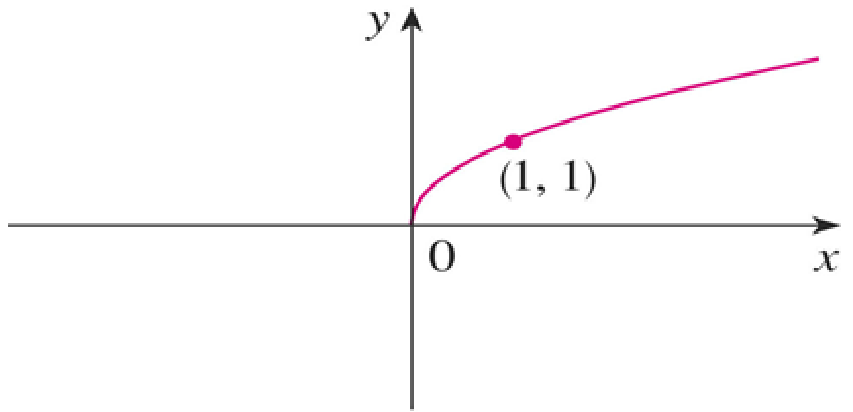




CASE

a = 1/*n*, where *n* is a positive integer

- The function $f(x) = x^{1/n} = \sqrt[n]{x}$ is a root function.
- For *n* = 2, it is the square root function $f(x) = \sqrt{x}$, whose domain is $[0, \infty)$ and whose graph is the upper half of the parabola $x = y^2$.
- For other even values $y = \sqrt[n]{x}$ of *n*, the graph of $y = \sqrt{x}$ is similar to that of .

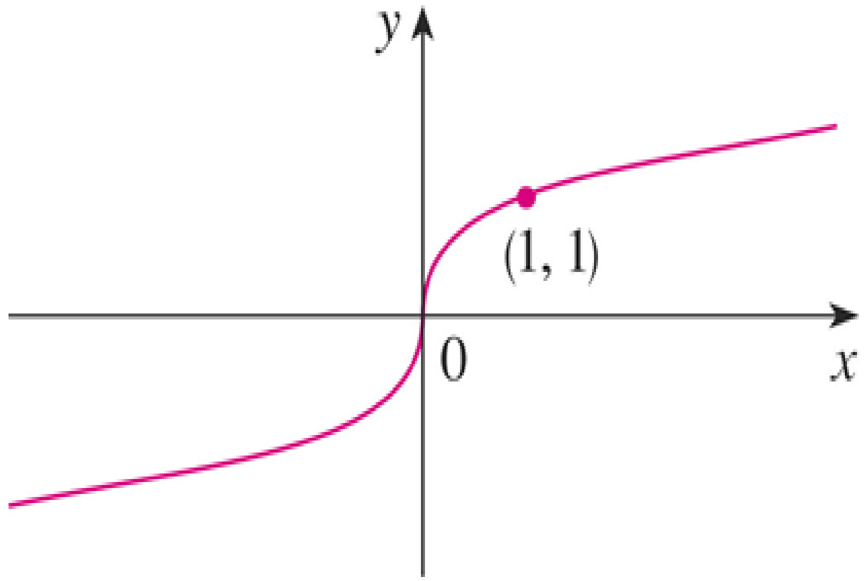


(a) $f(x) = \sqrt{x}$

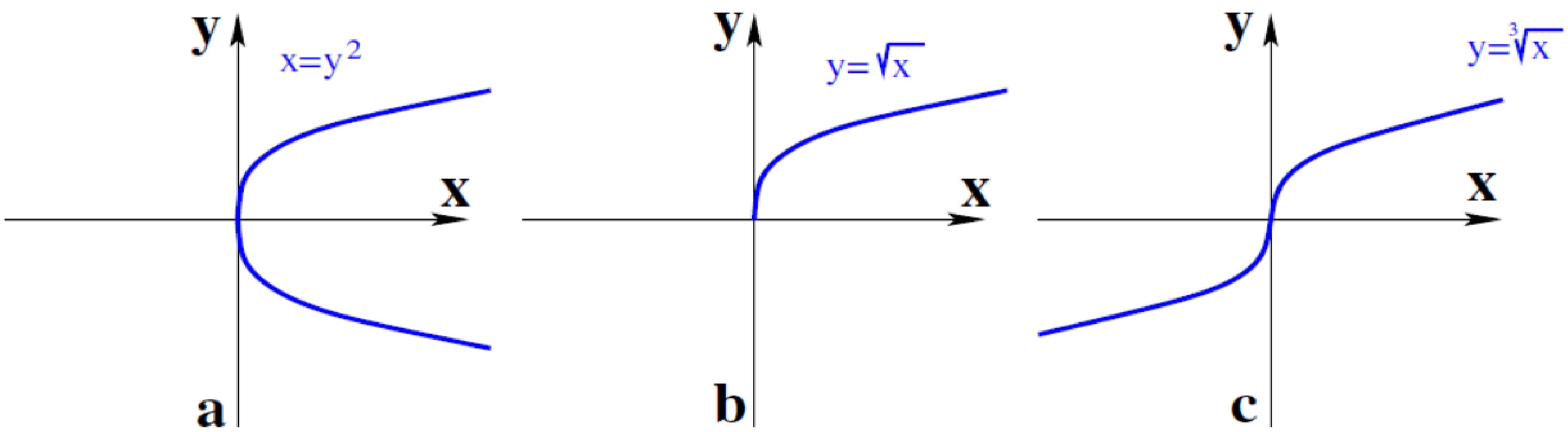
CASE

For *n* = 3, we have the cube root function $f(x) = \sqrt[3]{x}$ whose domain is (recall that every real number has a cube root) and whose graph is shown.

- The graph of $y = \sqrt[n]{x}$ for *n* odd (*n* > 3) is similar to that of $y = \sqrt[3]{x}$.



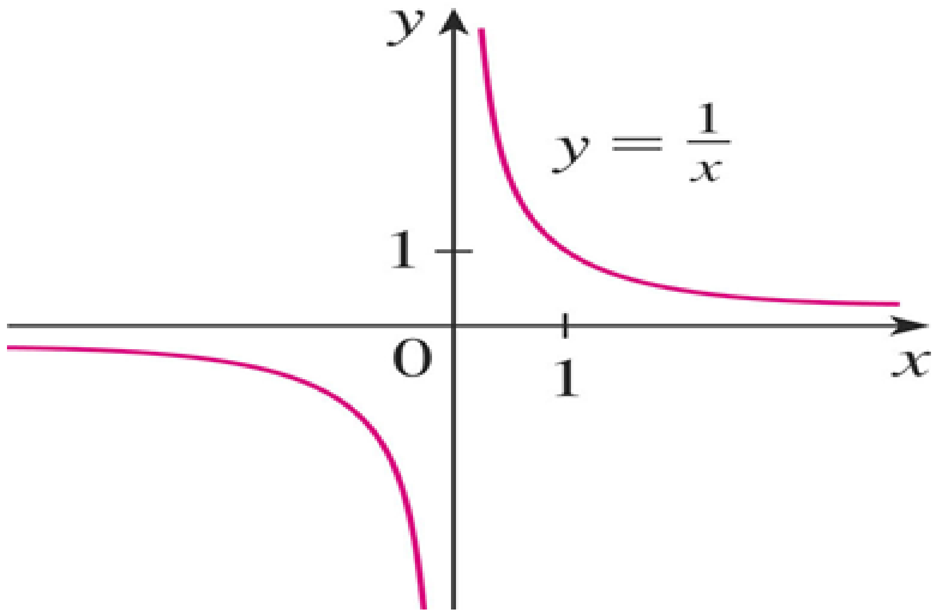
(b) $f(x) = \sqrt[3]{x}$


y
x=y²
x
a
y
y=√x
x
b
y
y=∛x
x
c

CASE

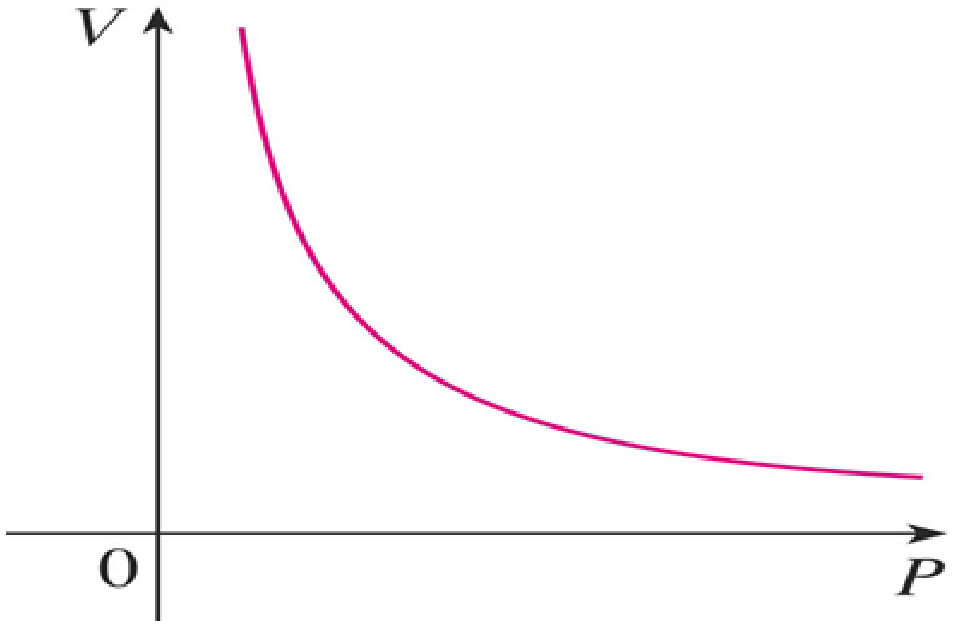
a = -1

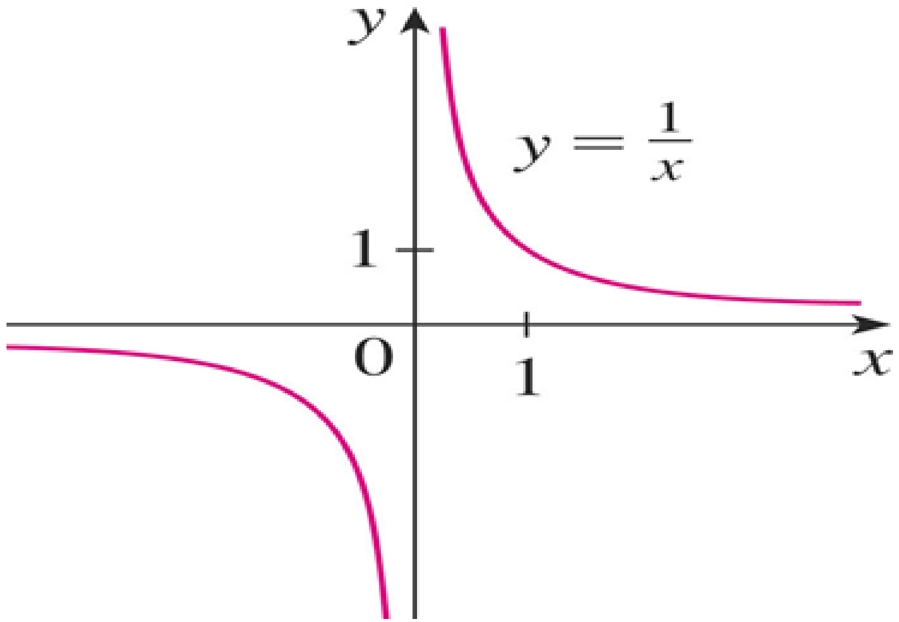
- The graph of the reciprocal function $f(x) = x^{-1} = 1/x$ is shown.
- Its graph has the equation $y = 1/x$, or $xy = 1$.
- It is a hyperbola with the coordinate axes as its asymptotes.



CASE

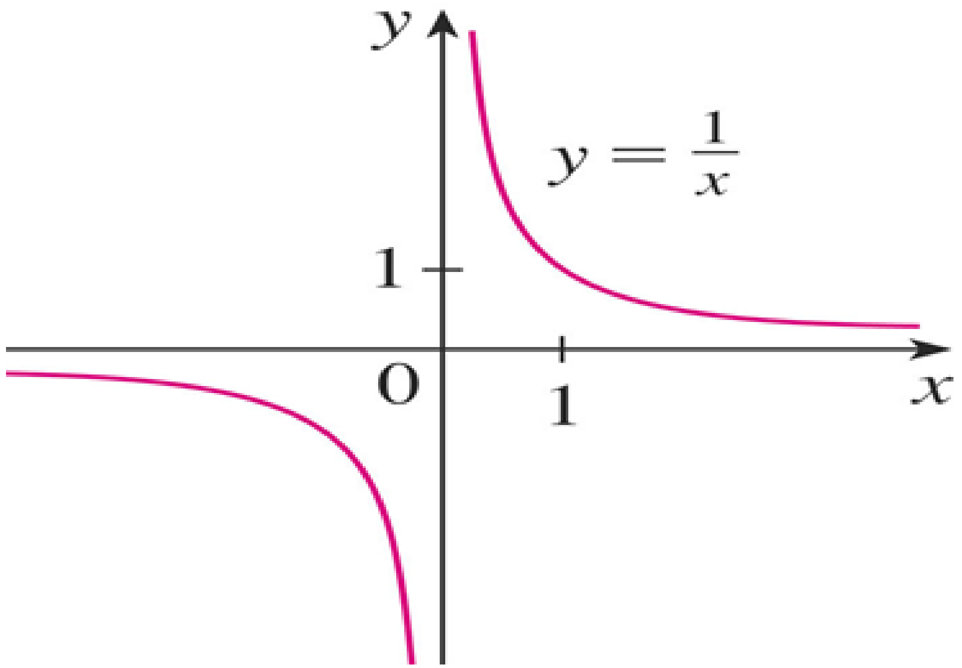
- This function arises in physics and chemistry in connection with Boyle's Law, which states that, when the temperature is constant, the volume *V* of a gas is inversely proportional to the pressure *P*. $V=C/P$
- where *C* is a constant. So, the graph of *V* as a function of *P* has the same general shape as the right half of the previous figure.





RATIONAL FUNCTIONS

- A rational function *f* is a ratio of two polynomials $f(\mathrm{x}) = \frac{P(x)}{Q(x)}$ where *P* and *Q* are polynomials. The domain consists of all values of *x* such that Q(x)≠0.
- A simple example of a rational function is the function *f*(*x*) = 1/*x*, whose domain is $\{\mathrm{x}|\mathrm{x} \neq 0\}$.
- This is the reciprocal function graphed in the figure.

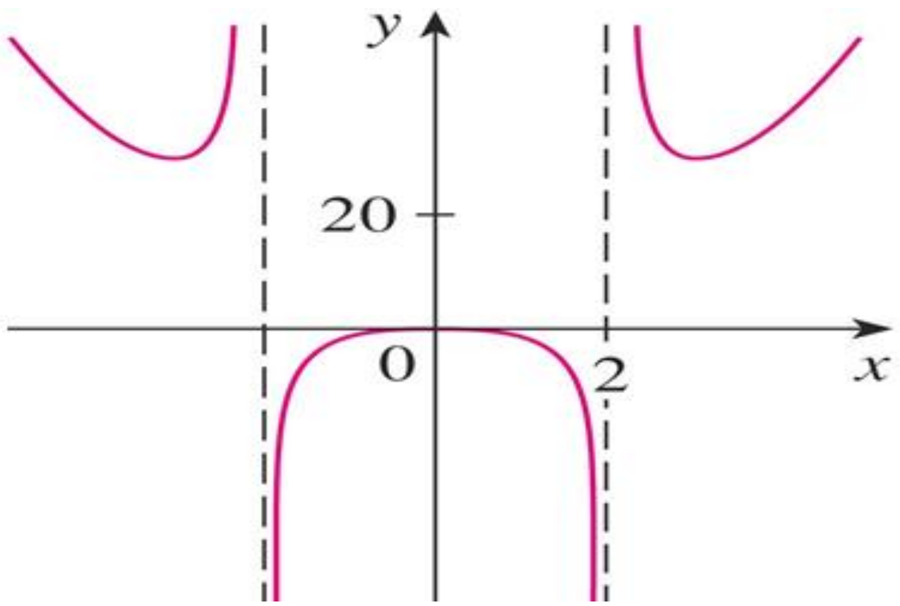


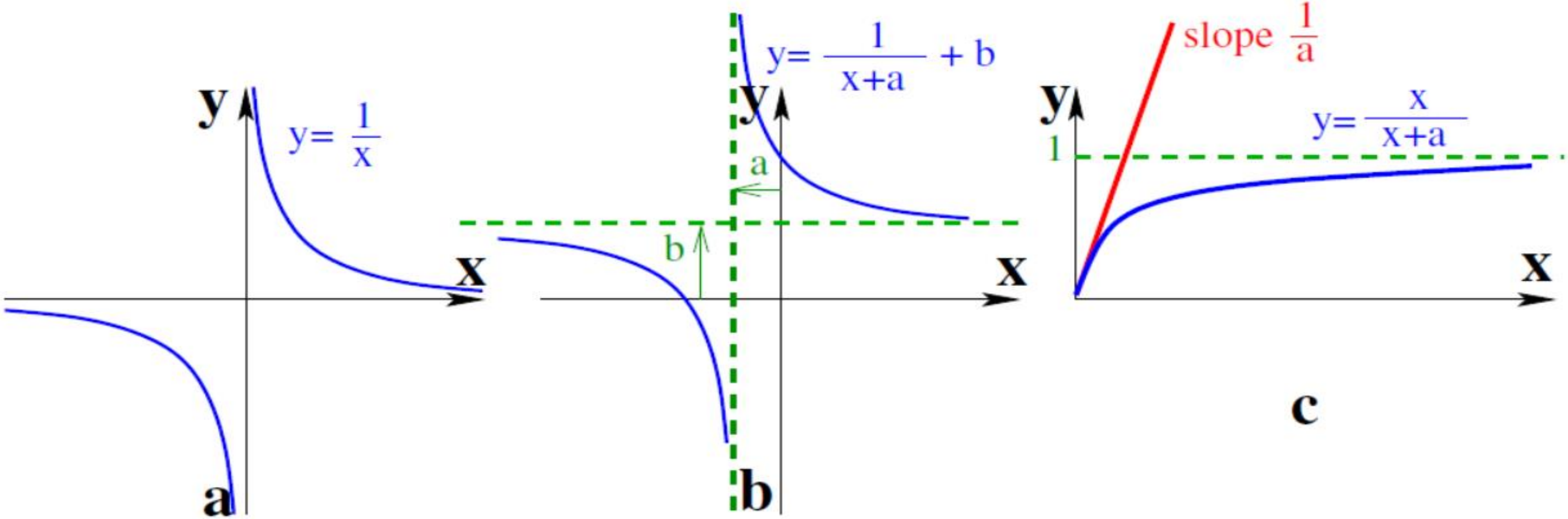
RATIONAL FUNCTIONS

The function is a rational function with domain.

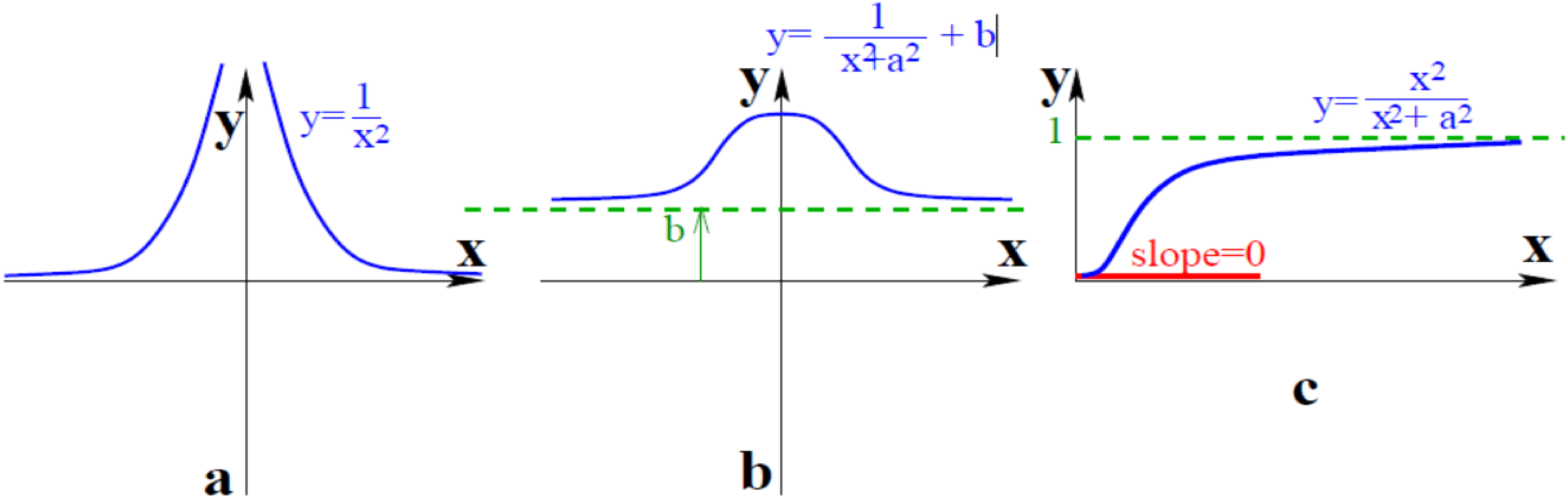
$$\{x \mid x \neq \pm 2\}$$

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$$





y
x
y= 1/x
a
y= 1/(x+a) + b
a
b
b
slope 1/a
y= x/(x+a)
1
c



y
$y=\frac{1}{x^2}$
x
a
y
$y=\frac{1}{x^2+a^2}+b$
b
x
b
y
$y=\frac{x^2}{x^2+a^2}$
1
slope=0
x
c

ALGEBRAIC FUNCTIONS

A function *f* is called an algebraic function if it can be constructed using algebraic operations—such as addition, subtraction, multiplication, division, and taking roots—starting with polynomials.

Any rational function is automatically an algebraic function.

Here are two more examples:

$$f(x) = \sqrt{x^2 + 1}$$

$$g(x) \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$$

ALGEBRAIC FUNCTIONS

An example of an algebraic function occurs in the theory of relativity.

– The mass of a particle with velocity *v* is

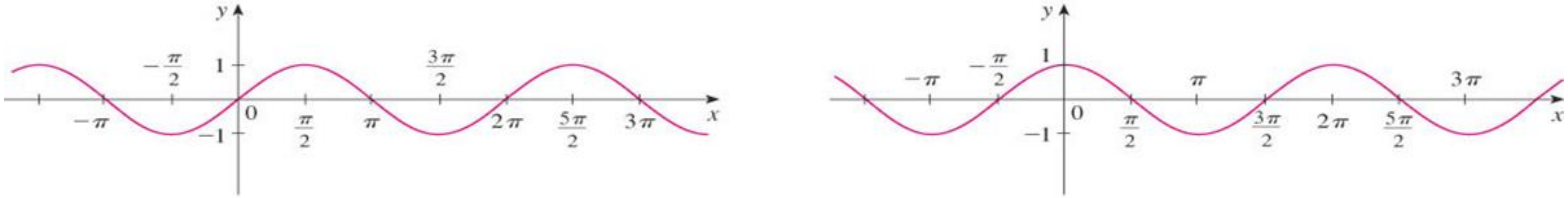
$$m = f(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the rest mass of the particle and $c = 3.0 \times 10^5$ km/s is the speed of light in a vacuum.

TRIGONOMETRIC FUNCTIONS

In calculus, the convention is that radian measure is always used (except when otherwise indicated).

- For example, when we use the function $f(x) = \sin x$, it is understood that $\sin x$ means the sine of the angle whose radian measure is x.
- Thus, the graphs of the sine and cosine functions are as shown in the figure.



(a) $f(x) = \sin x$ (b) $g(x) = \cos x$

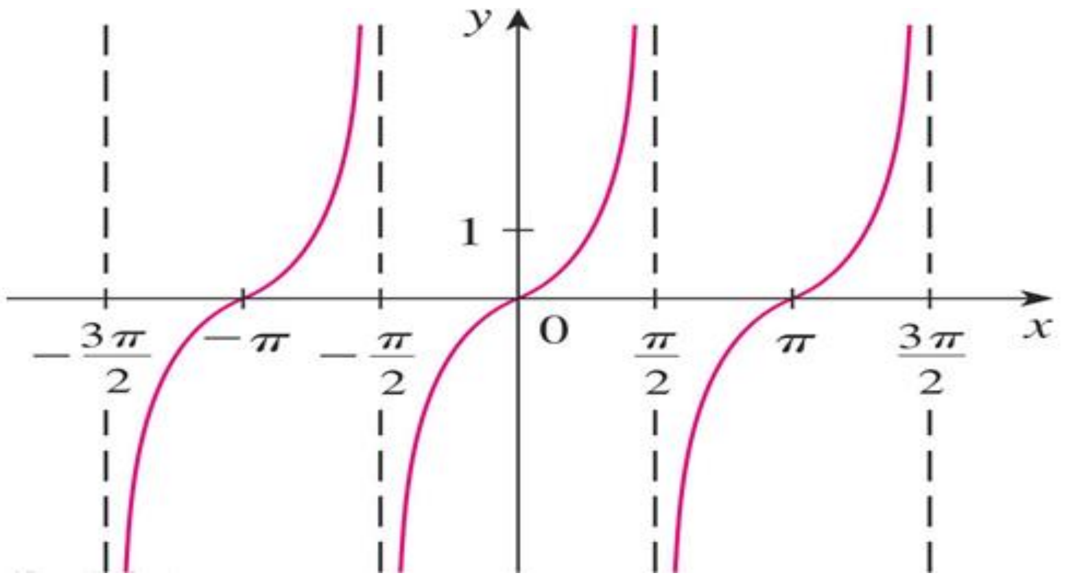
TRIGONOMETRIC FUNCTIONS

- Also, the zeros of the sine function occur at the integer multiples of π . That is, sin x = 0 when $x = n\pi$, n an integer.
- An important property of the sine and cosine functions is that they are periodic functions and have a period 2π. This means that, for all values of x, $\sin(x + 2\pi) = \sin(x)$, $\cos(x + 2\pi) = \cos(x)$.
- Notice that, for both the sine and cosine functions, the domain is $(-\infty, \infty)$ and the range is the closed interval [-1, 1]. Thus, for all values of x, we have: $-1 \leq \sin(x) \leq 1$, $-1 \leq \cos(x) \leq 1$. In terms of absolute values, it is: $|\sin(x) \leq 1|$, $|\cos(x) \leq 1|$.
- The periodic nature of these functions makes them suitable for modeling repetitive phenomena such as tides, vibrating springs, and sound waves.

$$L(t) = 12 + 2.8\sin\left[\frac{2\pi}{365}(t-80)\right]$$

TRIGONOMETRIC FUNCTIONS

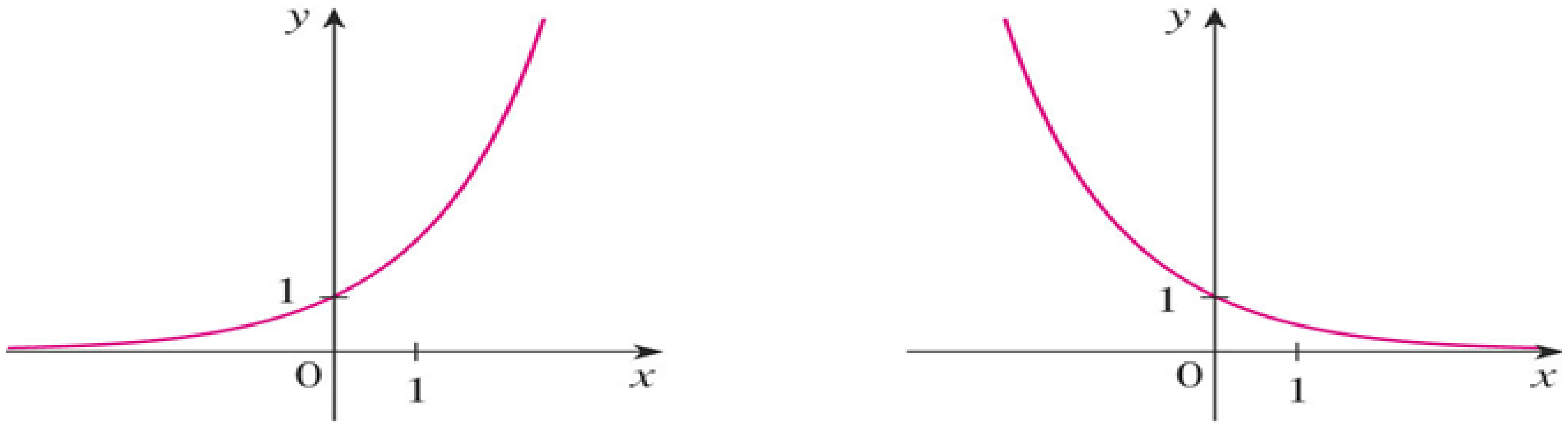
- The tangent function is related to the sine and cosine functions by the equation $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- The tangent function is undefined whenever cos x = 0, that is, when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
- Its range is $(-\infty, \infty)$. Notice that the tangent π function has period: tan(x+π) = tan(x) for all x.
- The remaining three trigonometric functions—cosecant, secant, and cotangent—are the reciprocals of the sine, cosine, and tangent functions.



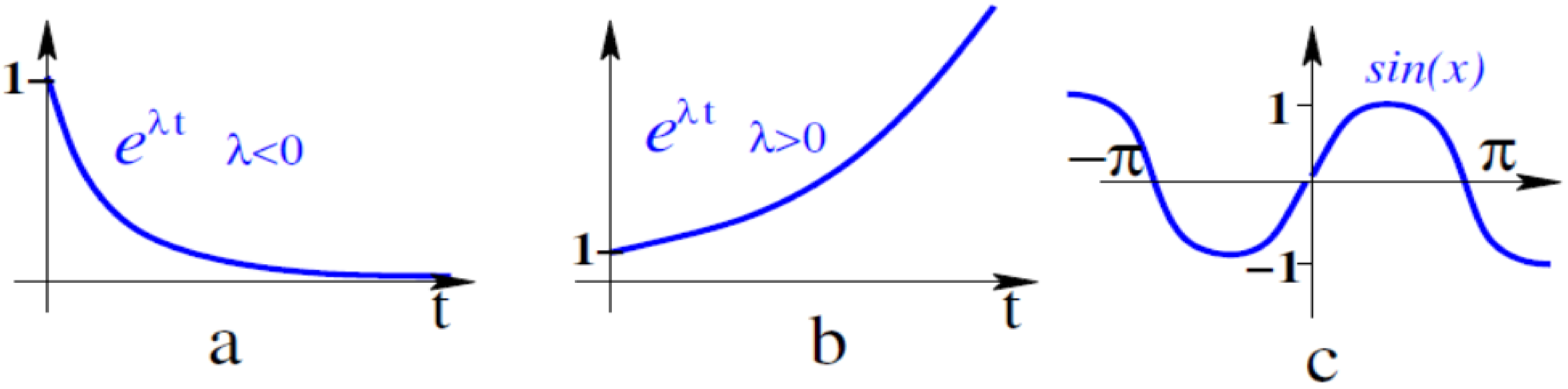
EXPONENTIAL FUNCTIONS

The exponential functions are the functions of the form $f(x)=a^x$, where the base *a* is a positive constant.

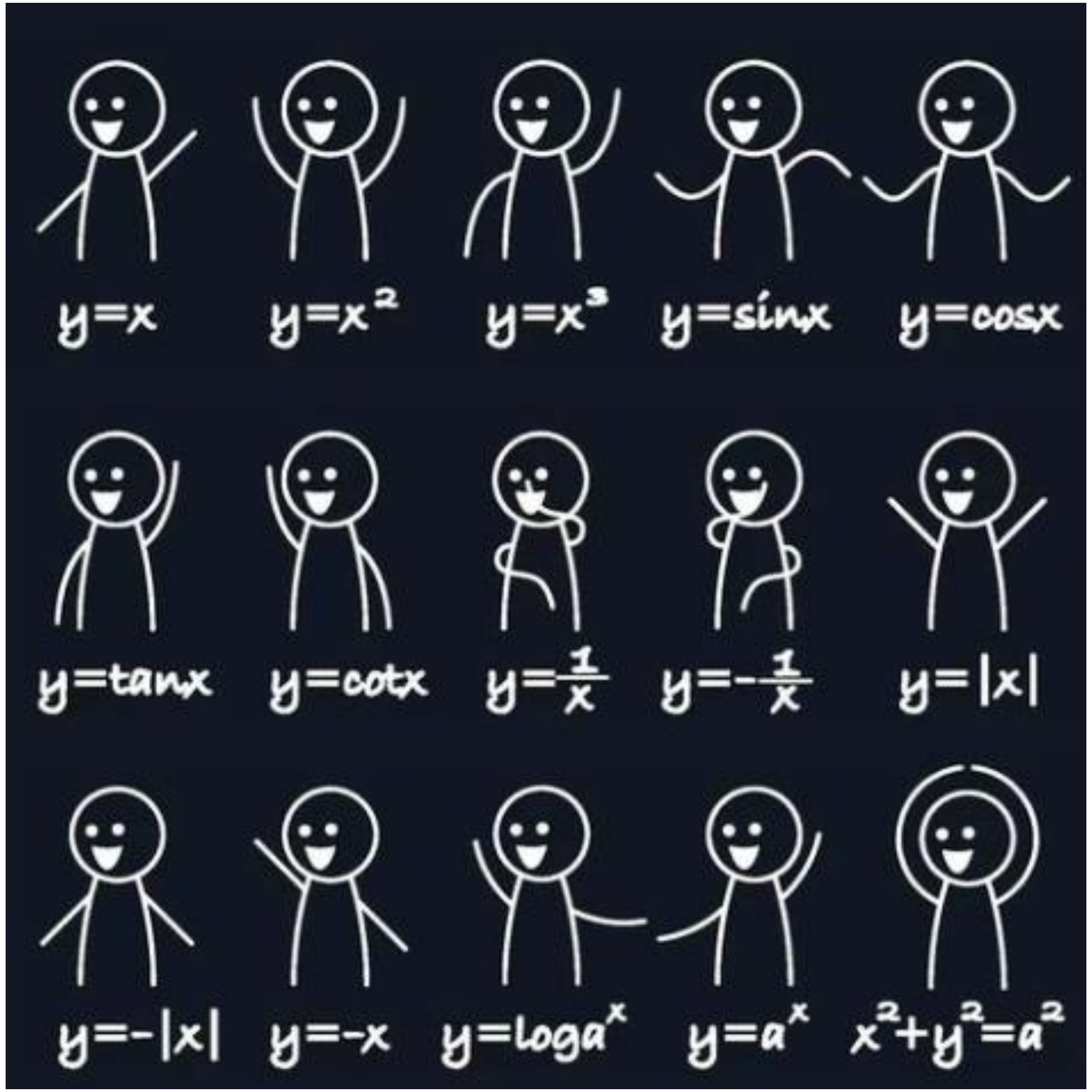
- The graphs of $y = 2^x$ and $y = (0.5)^x$ are shown.
- In both cases, the domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.
- We will see that they are useful for modeling many natural phenomena—such as population growth (if $a > 1$) and radioactive decay (if $a < 1$).
- The logarithmic functions $f(\mathrm{x}) = log_a$ x, where the base a is a positive constant, are the inverse functions of the exponential functions.



(a) $y = 2^x$ (b) $y = (0.5)^x$


1
$e^{\lambda t}$ $\lambda<0$
t
a
$e^{\lambda t}$ $\lambda>0$
1
t
b
sin(x)
1
$-\pi$
π
-1
c

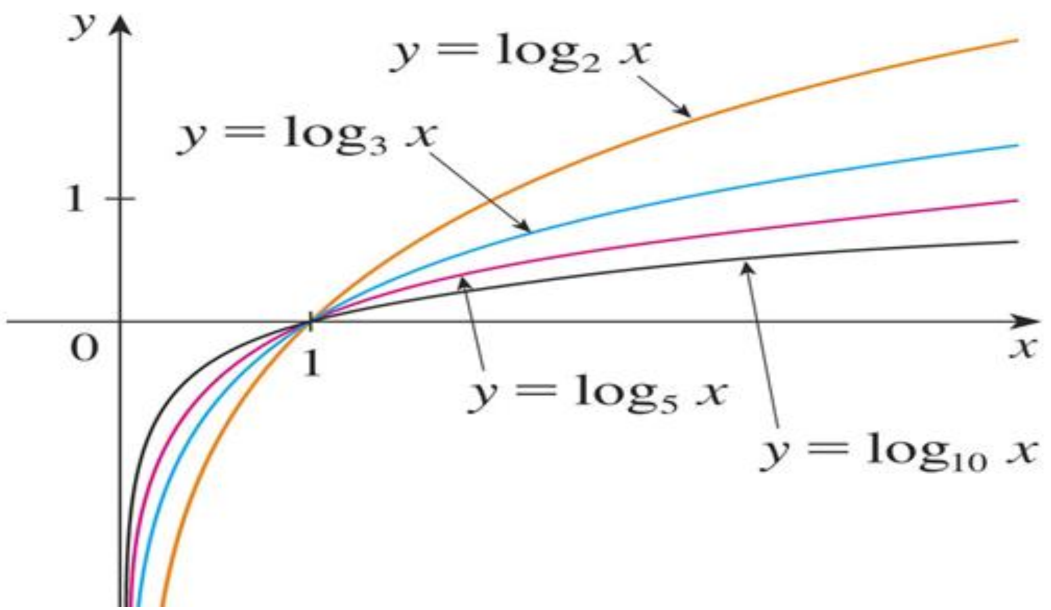
Davranışların Matematiksel Modellenmesi



LOGARITHMIC FUNCTIONS

The figure shows the graphs of four logarithmic functions with various bases.

– In each case, the domain is $(0, \infty)$, the range i $(-\infty, \infty)$, and the function increases slowly when x > 1



TRANSCENDENTAL FUNCTIONS

Classify the following functions as one of the types of functions that we have discussed.

- $f(x) = 5^x$ is an exponential function. The x is the exponent
- $g(x) = x^5$ is a power function. The x is the base. We could also consider it to be a polynomial of degree 5.
- $u(t) = 1 - t + 5t^4$ is a polynomial of degree 4.

$$h(x) = \frac{1+x}{1-\sqrt{x}}$$

This is an algebraic function.

Transcendental functions are those that are not algebraic.

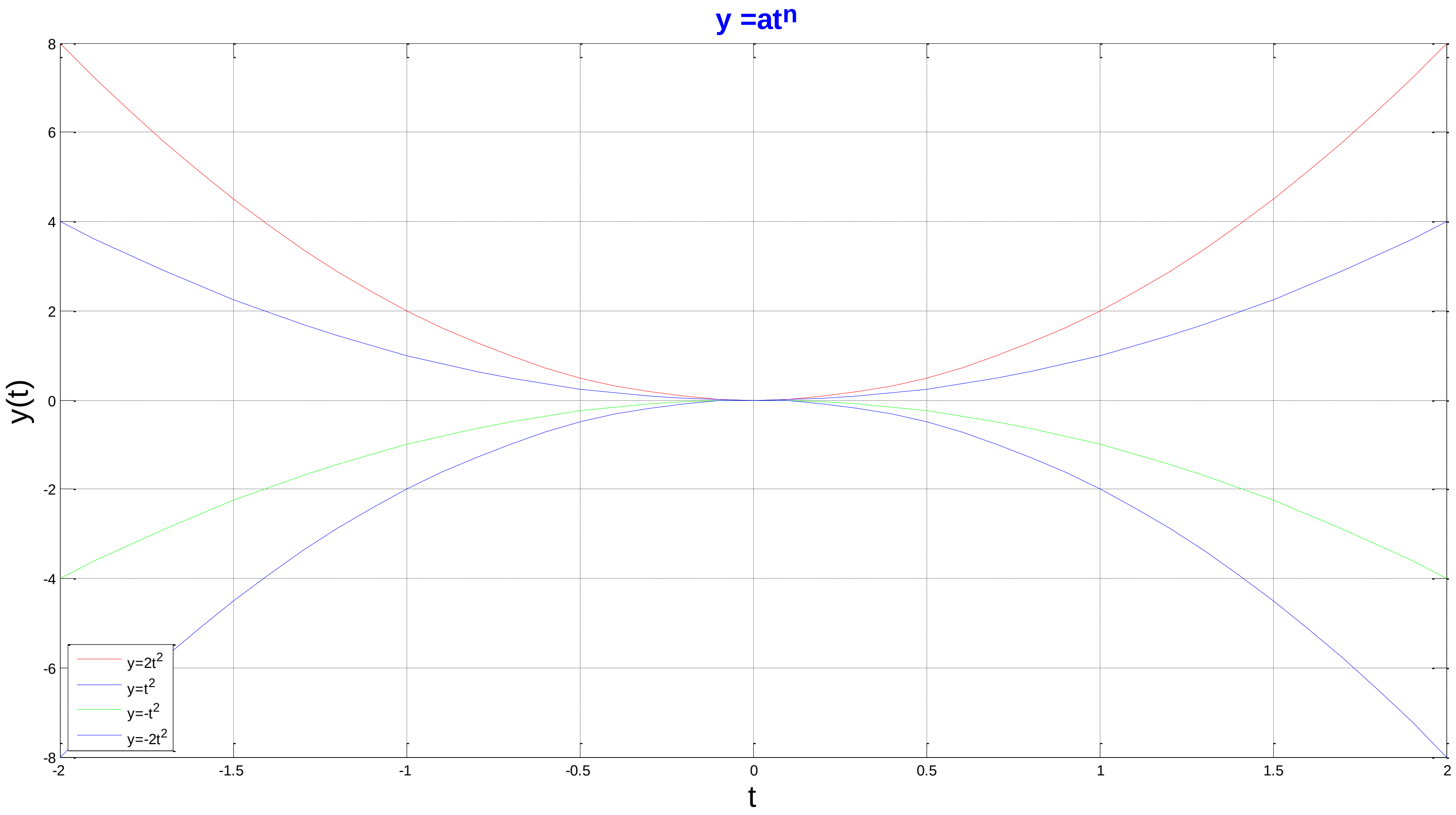
- The set of transcendental functions includes the trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- However, it also includes a vast number of other functions that have never been named.

Uygulama-1

```
•   clear all; close all
•   M=41; s=(M-1)/20;
•   for i=1:M
•       t(i)=-s+(i-1)*0.1;
•   end

•   for i=1:M
•       y1(i)=2*t(i)^2;
•       y2(i)=t(i)^2;
•       y3(i)=-t(i)^2;
•       y4(i)=-2*t(i)^2;
•   end

•   figure(1); plot(t,y1,'r',t,y2,'b',t,y3,'g',t,y4)
•   title('\fontsize{20}\bf y =at^{n}','Color','b')
•   xlabel('t','FontSize', 20)
•   ylabel('y(t)','FontSize', 20)
•   legend('y=2t^{2}','Location','southwest','y=t^{2}','Location','southwest','y=-
    t^{2}','Location','southwest','y=-2t^{2}','Location','southwest' )
•   grid on
```



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- If you choose to use them, I ask that you alert me of any mistakes which were made and allow me the option of incorporating such changes (with an acknowledgment) in my set of slides.

Sincerely,

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Kaynaklar

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- http://www.ee.cityu.edu.hk/~csl/sigana/sig01.ppt
- İnternet ortamından sunum ve ders notları

thank you